CAMOSUN

Name: SOLUTIONS

MATH 126 (Winter, 2015)
Term Test 1
by George Ballinger

Answer the questions in the space provided. This test has 17 questions for a total of 25 marks.

1. (2 marks) Construct a truth table for the proposition $\neg(p \wedge q) \rightarrow (p \vee q)$.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee q$	$\neg(p \wedge q) \rightarrow (p \vee q)$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	F

2. (1 mark) Which of the following propositions are tautologies? Circle your answer(s). No justification is required.

(i) $p \wedge p$ (ii) $p \vee p$ (iii) $p \oplus p$ (iv) $p \rightarrow p$ (v) $p \leftrightarrow p$ (vi) $\neg p$

3. (1 mark) Write the statement, "You will stay dry only if you use an umbrella," in the form, "If..., then ...."

If you stay dry, then you use an umbrella.

4. (1 mark) Write the contrapositive of the statement, "If you are snoring, then you are asleep."

If you are not asleep, then you are not snoring.

5. (1 mark) Which of the following propositions is logically equivalent to $(p \wedge q) \vee (\neg p \wedge \neg q)$? Circle your answer. No justification is required.

(i) $p \wedge q$ (ii) $p \vee q$ (iii) $p \oplus q$ (iv) $p \rightarrow q$ (v) $p \leftrightarrow q$ (vi) $\neg p$

6. (1 mark) Using simple English, write the negation of the statement, "I will climb the mountain or I will die trying." (Do not simply use words like "It is not the case that....")

I will not climb the mountain and I will not die trying.

7. (2 marks) Using only the logical equivalences listed on the "Logical Equivalences" supplement, prove $p \wedge \neg(\neg q \wedge p) \equiv p \wedge q$. At each step identify the logical equivalence being used.

$$\begin{aligned} p \wedge \neg(\neg q \wedge p) &\equiv p \wedge (\neg(\neg q) \vee \neg p) && \text{De Morgan} \\ &\equiv p \wedge (q \vee \neg p) && \text{double negation} \\ &\equiv (p \wedge q) \vee (p \wedge \neg p) && \text{distributive} \\ &\equiv (p \wedge q) \vee F && \text{negation} \\ &\equiv p \wedge q && \text{identity} \end{aligned}$$

8. (3 marks) Using a direct proof, prove that for any integers x and y, if x is even and $x + y$ is odd, then y is odd.

Let $x, y \in \mathbb{Z}$ and assume x is even and $x+y$ is odd.
Then $x = 2k$ and $x+y = 2j+1$ for some $k, j \in \mathbb{Z}$.
Thus $y = (x+y) - x = (2j+1) - 2k = 2(j-k)+1 = 2i+1$,
where $i = j-k \in \mathbb{Z}$, and so y is odd. ∎

9. (1 mark) Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain of each variable is the set of real numbers. Briefly justify your answer.

True. If $x = 0$ (or any negative real number), then $x \leq y^2$ for all $y \in \mathbb{R}$.

10. (1 mark) Define $Q(x)$ to be "x is a rational number," where the domain is the set of real numbers. Express the following in terms of $Q(x)$, quantifiers and logical connectives: "The sum of any two rational numbers is rational."

$\forall x \forall y \left((Q(x) \wedge Q(y)) \rightarrow Q(x+y) \right)$

11. (3 marks) Prove that for any real numbers $a_1, a_2, a_3, \ldots, a_n$, at least one of the numbers must be greater than or equal to their average $A = (a_1 + a_2 + \cdots + a_n)/n$.

Suppose, for the sake of contradiction, that the statement is false and that all n numbers are less than their average A. (i.e. $a_i < A$ for $i = 1, 2, \ldots, n$). Then

$$a_1 + a_2 + \ldots + a_n < \underbrace{A + A + \ldots + A}_{n \text{ of these}} = nA = a_1 + a_2 + \ldots + a_n,$$

which is a contradiction. Thus at least one of the numbers must be greater than or equal to their average. ■

12. (1 mark) If A and B are sets, then circle the expression that is equivalent to $\overline{B \cup \overline{A}}$. No justification is required.

$\overline{B \cup \overline{A}} = \overline{B} \cap \overline{\overline{A}} = \overline{B} \cap A$
$= A \cap \overline{B} = A - B$

(i) $A \oplus B$ (ii) $A - B$ (iii) $B - A$ (iv) $A \cup B$ (v) $A \cap B$

13. (1 mark) Circle all the statements listed below that are true, where $\varnothing$ denotes the empty set. No justification is required.

(i) $\varnothing \subseteq \{\varnothing\}$ (ii) $\varnothing \subset \{\varnothing\}$ (iii) $\{\varnothing\} \subseteq \{\varnothing\}$ (iv) $\varnothing \in \{\varnothing\}$ (v) $\{\varnothing\} \in \{\varnothing\}$

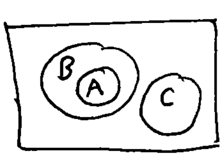
14. (1 mark) Find the power set of $\{a, b, c\}$.

$\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, b, c\}\}$

15. (1 mark) What can you say about the sets A and B if $A \cap B = A$?

$A \subseteq B$

16. (1 mark) Suppose A, B and C are finite sets with $|A| = 5$, $|B| = 10$ and $|C| = 15$. Find $|A \cup B \cup C|$ if $A \subseteq B$ and the sets B and C are disjoint.



$|A \cup B \cup C| = |B| + |C| = 10 + 15 = 25$

17. (3 marks) Consider the following argument.

If I do not do a good job, then I will be fired.
If I do a good job or I curry favour with my boss, I will be promoted and get a raise.
I did not get fired.

$\therefore$ I will get a raise.

(a) Define p, q, r, s and t as follows and rewrite the argument symbolically using logic notation.

p: I do a good job.
q: I get fired.
r: I curry favour with my boss.
s: I get promoted.
t: I get a raise.

$\neg p \to q$
$(p \vee r) \to (s \wedge t)$
$\neg q$

$\therefore t$

(b) Prove that the argument in part (a) is valid. State the reason for each step of your proof by identifying hypotheses or citing an appropriate rule of inference or logical equivalence.

STEP	REASON
1. $\neg q$	hyp.
2. $\neg p \to q$	hyp.
3. $\neg(\neg p)$	Modus Tollens (1,2)
4. p	double negation (3)
5. $p \vee r$	Addition (4)
6. $(p \vee r) \to (s \wedge t)$	hyp.
7. $s \wedge t$	Modus Ponens (5,6)
8. $t \wedge s$	Commutativity of $\wedge$ (7)
9. t	Simplification (8)