CAMOSUN

Name: SOLUTIONS

MATH 126 (Winter, 2018)

Term Test 1

by George Ballinger

Answer the questions in the space provided.
This test has 15 questions for a total of 40 marks.

1. (3 marks) Construct a truth table for the proposition $((p \wedge q) \to \neg p) \vee q$.

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \to \neg p$	$((p \wedge q) \to \neg p) \vee q$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	F	T	T	T
F	F	F	T	T	T

2. (2 marks) Find a proposition using only p, q, $\neg$ and $\vee$ (and if necessary, brackets) having the following truth table.

p	q	?
T	T	F
T	F	F
F	T	F
F	F	T

$\neg(p \vee q)$

3. (4 marks) Write the following statements in the form, "If..., then...."
   (a) You get dessert only if you eat your vegetables.

   If you get dessert, then you've eaten your vegetables.

   (b) Paying a high salary is necessary for attracting workers.

   If you attract workers, then you pay a high salary.

4. (2 marks) Write the inverse of the following junk removal company's slogan, "If you call, then we haul."

If you don't call, then we don't haul.

5. (2 marks) Which of the following statements are true, if the domain of each variable is the set of real numbers. Circle your answer(s). No justification is required.

(i) $\exists x \exists y (y = x^2)$ (iv) $\exists y \forall x (y = x^2)$

(ii) $\forall x \forall y (y = x^2)$ (v) $\forall x \exists y (y = x^2)$

(iii) $\exists x \forall y (y = x^2)$ (vi) $\forall y \exists x (y = x^2)$

6. (2 marks) Consider the statement,

"Every student answered question 5, yet some students did not get it right."

Let $A(x)$ be "x answered question 5" and $R(x)$ be "x got it right," where the domain is the set of all students. Express this statement symbolically in terms of $A(x)$, $R(x)$, quantifiers and logical connectives.

$$(\forall x\, A(x)) \wedge (\exists x\, \neg R(x))$$

OR $$\forall x \exists y (A(x) \wedge \neg R(y))$$

7. (1 mark) If using a proof by contraposition technique to prove a conditional statement $p \to q$, which one of the following assumptions is made about p or q? Circle your answer. No justification is required.

(i) p is true (ii) p is false (iii) q is true (iv) q is false

8. (5 marks) Prove the following by using only the logical equivalences listed on the "Logical Equivalences" supplement. At each step identify the logical equivalence rule being used.

$$\neg((\neg q \wedge p) \vee (p \wedge \neg p)) \equiv p \to q$$

$$\begin{aligned}
\neg((\neg q \wedge p) \vee (p \wedge \neg p)) &\equiv \neg((\neg q \wedge p) \vee F) && \text{Negation} \\
&\equiv \neg(\neg q \wedge p) && \text{Identity} \\
&\equiv \neg(\neg q) \vee \neg p && \text{De Morgan} \\
&\equiv q \vee \neg p && \text{Double Negation} \\
&\equiv \neg p \vee q && \text{Commutative} \\
&\equiv p \to q && \text{Equivalence of Implication}
\end{aligned}$$

9. (2 marks) Using simple English, write the negation of the statement,

"Donald Trump is 'like, really smart' or he is 'a very stable genius.'"

Do not simply use words like "It is not the case that...."

Donald Trump is not 'like, really smart' and he is not 'a very stable genius.'

10. (1 mark) What can you say about the sets A and B if $A - B = \varnothing$, where $\varnothing$ denotes the empty set?

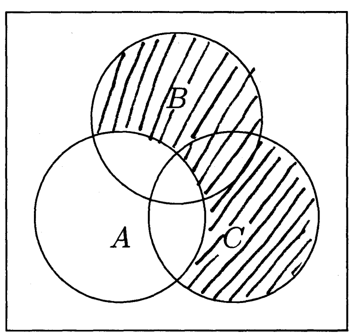
$A \subseteq B$

11. (2 marks) If $\mathcal{P}(A)$ denotes the power set of a finite set A, then find $\mathcal{P}(\mathcal{P}(\varnothing))$, where $\varnothing$ denotes the empty set.

$P(\phi) = \{\phi\}$

$\therefore P(P(\phi)) = \{\phi, \{\phi\}\}$

12. (1 mark) Shade the region of the Venn Diagram corresponding to $\overline{A} \cap (B \cup C)$.



13. (2 marks) Let A, B and C be sets. Prove that $A \cap B \subseteq B \cup C$. Recall that a Venn Diagram is not considered an acceptable method of proof.

Let $x \in A \cap B$. Then $x \in A$ and $x \in B$. (by def'n of $\cap$)
Since $x \in B$ then $x \in B$ or $x \in C$ and so $x \in B \cup C$ (by def'n of $\cup$).
Thus $A \cap B \subseteq B \cup C$ ■

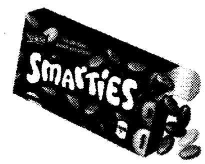
14. (6 marks) Consider the following argument.

If I suck Smarties very slowly, then I do not eat the red Smarties last.
I either suck Smarties very slowly or I crunch Smarties very fast.
I eat the red Smarties last and eating Smarties is a blast.

$\therefore$ I crunch Smarties very fast.

(a) Define p, q, r and s as follows and rewrite the argument symbolically using logic notation.

p: "I suck Smarties very slowly"
q: "I eat the red Smarties last"
r: "I crunch Smarties very fast"
s: "Eating Smarties is a blast"



$$\begin{array}{l} p \to \neg q \\ p \vee r \\ \underline{q \wedge s} \\ \therefore\ r \end{array}$$

(b) Prove that the argument in part (a) is valid. State the reason for each step of your proof by identifying hypotheses or citing an appropriate rule of inference or logical equivalence.

STEP	REASON
1. $q \wedge s$	hyp.
2. q	Simplification (1)
3. $\neg(\neg q)$	double negation (2)
4. $p \to \neg q$	hyp.
5. $\neg p$	Modus Tollens (3,4)
6. $p \vee r$	hyp
7. r	Disjunctive Syllogism (5,6)

15. (5 marks) Use a proof by contradiction to prove that if x is an irrational number and y is a rational number, then the difference $x - y$ is irrational.

Suppose x is irrational and y is rational, yet $x-y$ is not irrational, i.e. $x-y$ is rational. Since y and $x-y$ are rational then $y = \frac{a}{b}$ and $x-y = \frac{c}{d}$ for some $a, b, c, d \in \mathbb{Z}$ where $b \neq 0$ and $d \neq 0$.

Thus $x = (x-y) + y = \frac{c}{d} + \frac{a}{b} = \frac{bc+ad}{bd} = \frac{p}{q}$

where $p = bc+ad \in \mathbb{Z}$ and $q = bd \in \mathbb{Z}$ and $q \neq 0$, implying x is rational, which contradicts the fact that x is irrational, thus proving the statement. ■