Proof Exercises

Definitions

1. An integer n is **even** if $n = 2k$ for some integer k. An integer n is **odd** if $n = 2k + 1$ for some integer k.

2. A real number x is **rational** if $x = p/q$ for some integers p and q, where $q \neq 0$. A real number x is **irrational** if it is not rational.

Exercises

1. Prove that if n is an odd integer, then n^2 is an odd integer.

2. Prove that if x and y are rational numbers, then $x + y$ is rational.

3. Let $P(n)$ be the statement "if $n \geq 4$, then $2^n \geq n^2$," where the domain is the set of integers. Prove $P(3)$.

4. Let $P(n)$ be the statement "if m is even, then nm is even," where the domain is the set of integers. Prove $P(0)$.

5. Prove that if $a + b + c \geq 1$, where $a, b, c \in \mathbb{R}$, then $a \geq 1/3$ or $b \geq 1/3$ or $c \geq 1/3$.

6. Prove or disprove that if x and y are both irrational, then $x + y$ is irrational.

7. Prove or disprove that if x is rational and y is irrational, then $x + y$ is irrational.

8. Prove that $\sqrt{2}$ is irrational.

9. Prove that for any nonnegative real numbers x and y, $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$ if, and only if, $xy = 0$.

10. Prove that for any integer x, $x^2 + x + 1$ is odd.

11. Prove that if the name of a month has 5 or more characters, then a 4-letter word can be formed using those characters.

12. Prove that there are no integer solutions to the equation $x^2 + y^2 = 7$.

13. Prove that there is an integer solution to the equation $x^2 + y^2 = 13$.

14. Prove that there exist irrational numbers x and y such that x^y is rational.

15. Prove that if n is an odd integer, then n can be expressed uniquely as the sum of two consecutive integers.