Permutations and Combinations

Permutations

Given a set of n distinct objects, a **permutation** is an *ordered* arrangement of these n objects.

An r**-permutation** (where $0 \le r \le n$) is an ordered arrangement of r elements of the set.

The number of such r-permutations is denoted by $P(n, r)$ or nPr (as on the Sharp EL-531 calculator) and is given by

$$\boxed{P(n,r) = \frac{n!}{(n-r)!}}$$

Special Cases:

$P(n,n) = n!$	There are $n!$ permutations of all n objects.
$P(n,1) = n$	There are n ordered lists containing one of the n objects.
$P(n,0) = 1$	There is one ordered list containing none of the n objects, namely the empty list.

If $r \ge 1$, then

$$\boxed{P(n,r) = n(n-1)(n-2)\cdots(n-r+1)}$$

Combinations

Given a set of n distinct objects, an r**-combination** (where $0 \le r \le n$) is an *unordered* selection of r of these n objects (i.e. it is a subset with r elements).

The number of such r-combinations is denoted by $C(n, r)$, nCr, or $\binom{n}{r}$ and is pronounced "n choose r" and is called a **binomial coefficient** (see sec 6.4). It is given by

$$\boxed{C(n,r) = \frac{n!}{r!(n-r)!}}$$

Special Cases:

$C(n,n) = 1$	There is one way of choosing all n of the n objects.
$C(n,1) = n$	There are n ways of choosing any one of the n objects.
$C(n,0) = 1$	There is one way of choosing none of the n objects, corresponding to the empty set.

Since selecting r objects from a set of n objects is equivalent to choosing the remaining $n-r$ objects to not be part of the selection, we get the following relationship

$$\boxed{C(n,r) = C(n,n-r)}$$

Finally, we note the relationship between counting permutations and combinations,

$$\boxed{P(n,r) = C(n,r)\cdot r!} \qquad \text{or equivalently} \qquad \boxed{C(n,r) = \frac{P(n,r)}{r!}}$$