

# Rules of Inference

## 1. Modus Ponens

$$\frac{p \quad p \rightarrow q}{\therefore q}$$

## 2. Modus Tollens

$$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$$

## 3. Hypothetical Syllogism

$$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$$

## 4. Disjunctive Syllogism

$$\frac{p \vee q \quad \neg p}{\therefore q}$$

## 5. Addition

$$\frac{p}{\therefore p \vee q}$$

## 6. Simplification

$$\frac{p \wedge q}{\therefore p}$$

## 7. Conjunction

$$\frac{p \quad q}{\therefore p \wedge q}$$

## 8. Resolution

$$\frac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$$

## Rules for Quantified Statements

### 1. Universal Instantiation

$$\frac{\forall x P(x)}{\therefore P(c)}$$

### 2. Universal Generalization

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

### 3. Existential Instantiation

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

### 4. Existential Generalization

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$