Mathematical Induction

Let $P(n)$ be a propositional function of a positive integer n. The following arguments are all valid (by using Modus Ponens and conjunction repeatedly).

$$\begin{array}{ll} & P(1) \\ \hline \therefore & P(1) \end{array}$$

$$\begin{array}{ll} & P(1) \\ & P(1) \to P(2) \\ \hline \therefore & P(1) \wedge P(2) \end{array}$$

$$\begin{array}{ll} & P(1) \\ & P(1) \to P(2) \\ & P(2) \to P(3) \\ \hline \therefore & P(1) \wedge P(2) \wedge P(3) \end{array}$$

$$\begin{array}{ll} & P(1) \\ & P(1) \to P(2) \\ & P(2) \to P(3) \\ & \quad\vdots \\ & P(k) \to P(k+1) \\ \hline \therefore & P(1) \wedge P(2) \wedge P(3) \wedge \cdots \wedge P(k) \wedge P(k+1) \end{array}$$

The Principle of Mathematical Induction (PMI) essentially says that the following "argument" is valid.[1]

$$\begin{array}{ll} & P(1) \\ & P(1) \to P(2) \\ & P(2) \to P(3) \\ & \quad\vdots \\ & P(k) \to P(k+1) \\ & \quad\vdots \\ \hline \therefore & P(1) \wedge P(2) \wedge P(3) \wedge \cdots \wedge P(k) \wedge P(k+1) \wedge \cdots \end{array}$$

PMI is therefore a rule of inference that asserts

$$[P(1) \wedge \forall k \in \mathbb{Z}^+ (P(k) \to P(k+1))] \to \forall n \in \mathbb{Z}^+ P(n).$$

[1] Strictly speaking this is not an argument as we defined it in sec 1.6 since the conclusion follows from an infinite number of premises.