Conditional Statements

Definition If p and q are propositions, then $p \to q$ is called a **conditional statement** (or an **implication**) and is read "if p, then q". The statement is true if both p and q are true, if p is false and q is true, or if both p and q are false. It is false only if p is true and q is false.

The truth table for $\to$ is given by

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

To better understand this truth table, define p and q as follows:

p : You cook dinner.

q : I take out the garbage.

The sentence, "If you cook dinner, then I'll take out the garbage" is symbolized by $p \to q$. Consider under what circumstances a person making this statement would be deemed to have lied.

In a conditional statement p is called the **hypothesis** and q is called the **conclusion**.

Note that if q is true, then so is $p \to q$, regardless of the truth value of p. Similarly if p is false, then $p \to q$ is again true, regardless of the truth value of q.

In the English language there are many equivalent ways of translating the conditional statement $p \to q$ as in the table below.

Translation of $p \to q$	Example
if p, then q	If you cook dinner, then I'll take out the garbage.
if p, q	If you cook dinner, I'll take out the garbage.
p implies q	You cooking dinner implies I'll take out the garbage.
p only if q	You will cook dinner only if I take out the garbage.
p is sufficient for q	You cooking dinner is sufficient for me to take out the garbage.
q, if p	I'll take out the garbage if you cook dinner.
q when p	I'll take out the garbage when you cook dinner.
q whenever p	I'll take out the garbage whenever you cook dinner.
q is necessary for p	My taking out the garbage is necessary for you to cook dinner.
q unless $\neg p$	I'll take out the garbage unless you don't cook dinner.
q follows from p	My taking out the garbage follows from you cooking dinner.

Definition Given an implication $p \to q$, its **converse** is $q \to p$, its **contrapositive** is $\neg q \to \neg p$, and its **inverse** is $\neg p \to \neg q$.

Definition with respect to $p \to q$	Symbolic Statement	Example
Implication	$p \to q$	If you cook dinner, then I'll take out the garbage.
Converse	$q \to p$	If I take out the garbage, then you'll cook dinner.
Contrapositive	$\neg q \to \neg p$	If I don't take out the garbage, then you won't cook dinner.
Inverse	$\neg p \to \neg q$	If you don't cook dinner, then I won't take out the garbage.

Truth Table:

p	q	$\neg p$	$\neg q$	Implication $p \to q$	Converse $q \to p$	Contrapositive $\neg q \to \neg p$	Inverse $\neg p \to \neg q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

Both $p \to q$ and $\neg q \to \neg p$ are **logically equivalent**; they have the same truth table values. Likewise both $q \to p$ and $\neg p \to \neg q$ are logically equivalent.

Definition If p and q are propositions, then $p \leftrightarrow q$ is called a **biconditional statement** (or **bi-implication**) and is read "p if, and only if, q". The statement is true if both p and q are true or if both p and q are false and it is false otherwise.

The truth table for $\leftrightarrow$ is given by

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

For example, you will get an A+ if, and only if, your course mark is at least 90%. The words "if, and only if" are sometimes abbreviated "iff". Other translations of $p \leftrightarrow q$ are "if p, then q and conversely" and "p is necessary and sufficient for q".

A bi-implication $p \leftrightarrow q$ is logically equivalent to $(p \to q) \wedge (q \to p)$, in other words p implies q and q implies p.