Basic Graph Terminology

George Ballinger

A **graph**[*] (or **undirected graph**) G is an ordered pair $G = (V, E)$, where

- V is a nonempty set of elements called **vertices** (or **nodes**), and
- E is a (possibly empty) set of elements called **edges**, where each edge e is an unordered pair $\{u, v\}$ (or the singleton $\{v\}$ if $u = v$) of vertices called its **endpoints**, which are said to **connect** the vertices. Vertices connected by an edge are said to be **adjacent** and the edge connecting them is said to be **incident with** those vertices.

A graph $G = (V, E)$ is called a **finite graph** if V and E are finite sets (otherwise it is called an **infinite graph**).

A **loop** is an edge $\{v\}$ connecting a vertex v to itself.

A graph $G = (V, E)$ without loops in which multiple edges may connect the same vertices is called a **multigraph**. In this case E is technically a multiset (it may have repeated elements) rather than a set. If m edges connect two vertices u and v, then this collection of edges may be referred to as an edge $\{u, v\}$ of **multiplicity** m.

A **simple graph** is a graph that does not contain multiple edges or loops.

A **pseudograph** is a graph that may contain multiple edges and loops.

A **directed graph** (or **digraph**) G is an ordered pair $G = (V, E)$, where

- V is a nonempty set of elements called **vertices** (or **nodes**), and
- E is a (possibly empty) set of elements called **directed edges** (or **arcs**), where each edge e is an ordered pair (u, v) of vertices, where the edge is directed from the **initial vertex** u to the **terminal vertex** (or **end vertex**) v. In this case u is said to be **adjacent to** v and v is said to be **adjacent from** u.

A directed graph $G = (V, E)$ in which multiple directed edges may connect from one vertex to another (possibly the same) vertex is called a **directed multigraph**. In this case E is a multiset. If m directed edges connect from one vertex u to another (possibly the same) vertex v, then this collection of directed edges may be referred to as a directed edge (u, v) of **multiplicity** m.

A **simple directed graph** is a directed graph that does not contain multiple directed edges or loops.

A **mixed graph** is a graph that may contain both directed and undirected edges.

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[*] The term *graph* may be used generically to refer to any graph, whether it has loops or not, whether it has directed edges or not, and whether it has multiple edges or not.

Type	Edges	Multiple Edges Allowed?	Loops Allowed?
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed & undirected	Yes	Yes

In an undirected graph the **degree of a vertex** v, denoted $\deg(v)$, is the number of edges incident with it, except that loops count twice toward the degree.

A vertex in an undirected graph is said to be **isolated** if it has degree zero and **pendant** if it has degree one.

In a directed graph the **in-degree of a vertex** v, denoted $\deg^{-}(v)$, is the number of edges having v as their terminal vertex. The **out-degree of a vertex** v, denoted $\deg^{+}(v)$, is the number of edges having v as their initial vertex.

A simple graph having n vertices in which every pair of vertices is connected by an edge is called the **complete graph on n vertices** and is denoted K_n.

A simple graph $G = (V, E)$ is called **bipartite** if V is the disjoint union of two nonempty sets V_1 and V_2 and every edge in E connects a vertex in V_1 with a vertex in V_2. The graph is called the **complete bipartite graph** and is denoted $K_{m,n}$ if V_1 has m vertices, V_2 has n vertices, and every vertex in V_1 is connected to every vertex in V_2.

A **subgraph** of a graph $G = (V, E)$ is a graph $H = (W, F)$ where $W \subseteq V$ and $F \subseteq E$. If $H \neq G$, then H is called a **proper subgraph** of G.

The **union** of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, which is denoted $G_1 \cup G_2$, is the simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$.