Asymptotic Notation

Definition of Big-O: Let f and g be functions from $\mathbb{R}$ (or $\mathbb{Z}$) to $\mathbb{R}$. We say that $f(x)$ is $O(g(x))$ if there are constants $C > 0$ and $k > 0$ (called *witnesses*) such that

$$|f(x)| \leq C|g(x)|, \quad \text{for all } x > k.$$

Note: If the domain is $\mathbb{Z}$, then we usually replace the variable x with n.

Symbolically, for $f(x)$ to be $O(g(x))$ it means

$$\exists C > 0\, \exists k > 0\, \forall x((x > k) \rightarrow (|f(x)| \leq C|g(x)|)).$$

Consequently, for $f(x)$ to **not** be $O(g(x))$ it means

$$\forall C > 0\, \forall k > 0\, \exists x((x > k) \wedge (|f(x)| > C|g(x)|)).$$

Definition of Big-Ω: Let f and g be functions from $\mathbb{R}$ (or $\mathbb{Z}$) to $\mathbb{R}$. We say that $f(x)$ is $\Omega(g(x))$ if there are constants $C > 0$ and $k > 0$ such that

$$|f(x)| \geq C|g(x)|, \quad \text{for all } x > k.$$

Definition of Big-Θ: Let f and g be functions from $\mathbb{R}$ (or $\mathbb{Z}$) to $\mathbb{R}$. We say that $f(x)$ is $\Theta(g(x))$ if $f(x)$ is both $O(g(x))$ and $\Omega(g(x))$.