

Classic Open Problems in Number Theory

as of February 2017

Largest Known Prime Number: Can you find a prime number greater than all known primes?

There are infinitely many prime numbers. As such, there is no largest prime number. The probability that a large arbitrary integer n is prime is approximately $1/\ln(n)$. As the size of integers increase, the probability that they are prime decreases; for example, the probability that a 500 digit integer is prime is about $1/\ln(10^{500}) \approx 0.087\%$. There is a perpetual search for ever larger prime numbers.

The largest known prime number (a Mersenne prime found on January 7, 2016) is: $2^{74,207,281} - 1$
It has 22,338,618 decimal digits.

The second largest known prime number (also a Mersenne prime), with over 17 million decimal digits, was found in 2013.

A prize of \$150,000 is available to the first individual or group who discovers a prime with at least 100 million digits. A prize of \$250,000 goes to the discoverer of a prime with at least a billion digits.

Twin Prime Conjecture: There are infinitely many primes p such that $p + 2$ is also prime.

Examples of twin primes: (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43)

The largest known twin primes (found on September 14, 2016) are: $2,996,863,034,895 \cdot 2^{1,290,000} \pm 1$
They have 388,342 decimal digits.

Even though primes get increasingly rare as you go further out along the number line, if this conjecture is true, then you will never stop finding pairs of primes that differ by 2. In April 2013 it was proven that you will never stop finding pairs of primes whose difference is at most 70,000,000. In April 2014 that bound was reduced to 246.

Goldbach's Conjecture: Every even integer $n > 2$ is the sum of two primes.

Examples:

$$4 = 2 + 2$$

$$6 = 3 + 3$$

$$8 = 3 + 5$$

$$10 = 3 + 7 \text{ or } 5 + 5$$

$$12 = 5 + 7$$

$$14 = 3 + 11 \text{ or } 7 + 7$$

A \$1,000,000 prize was offered between March 20, 2000 and March 20, 2002 to anyone who proved Goldbach's conjecture, but the prize went unclaimed and the conjecture remains open.