

Floor and Ceiling Functions

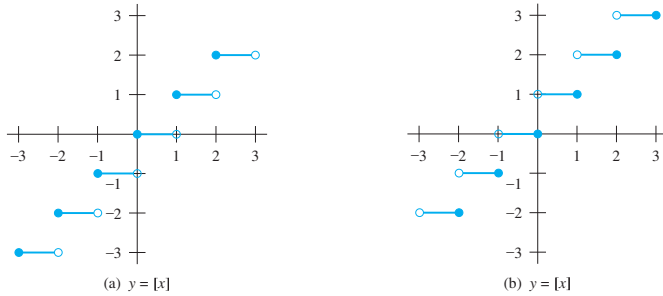


FIGURE 10 Graphs of the (a) Floor and (b) Ceiling Functions.

TABLE 1 Useful Properties of the Floor and Ceiling Functions.

(n is an integer, x is a real number)

- (1a) $\lfloor x \rfloor = n$ if and only if $n \leq x < n + 1$
- (1b) $\lceil x \rceil = n$ if and only if $n - 1 < x \leq n$
- (1c) $\lfloor x \rfloor = n$ if and only if $x - 1 < n \leq x$
- (1d) $\lceil x \rceil = n$ if and only if $x \leq n < x + 1$

(2) $x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$

(3a) $\lfloor -x \rfloor = -\lceil x \rceil$

(3b) $\lceil -x \rceil = -\lfloor x \rfloor$

(4a) $\lfloor x + n \rfloor = \lfloor x \rfloor + n$

(4b) $\lceil x + n \rceil = \lceil x \rceil + n$