Example of a Proof Using Logical Equivalences

Use Logical Equivalences to prove that $[(p \wedge \neg(\neg p \vee q)) \vee (p \wedge q)] \rightarrow p$ is a tautology.

Proof:

$$
\begin{aligned}
[(p \wedge \neg(\neg p \vee q)) \vee (p \wedge q)] \rightarrow p &\equiv [(p \wedge (\neg(\neg p) \wedge \neg q)) \vee (p \wedge q)] \rightarrow p && \text{De Morgan's law} \\
&\equiv [(p \wedge (p \wedge \neg q)) \vee (p \wedge q)] \rightarrow p && \text{Double Negation law} \\
&\equiv [((p \wedge p) \wedge \neg q) \vee (p \wedge q)] \rightarrow p && \text{Associative law} \\
&\equiv [(p \wedge \neg q) \vee (p \wedge q)] \rightarrow p && \text{Idempotent law} \\
&\equiv [p \wedge (\neg q \vee q)] \rightarrow p && \text{Distributive law} \\
&\equiv [p \wedge (q \vee \neg q)] \rightarrow p && \text{Commutative law} \\
&\equiv [p \wedge \mathbf{T}] \rightarrow p && \text{Negation law} \\
&\equiv p \rightarrow p && \text{Identity law} \\
&\equiv \neg p \vee p && \text{Equivalence of Implication} \\
&\equiv p \vee \neg p && \text{Commutative law} \\
&\equiv \mathbf{T} && \text{Negation law}
\end{aligned}
$$