Euler and Hamilton Paths and Circuits

Recall that paths and circuits are said to be simple if they do not contain the same edge more than once.

Definition 1: An Euler path in a graph G is a simple path containing every edge of G.

Definition 2: An Euler circuit in a graph G is a simple circuit containing every edge of G.

Definition 3: A Hamilton path in a graph G is a simple path that passes through every vertex of G exactly once.

Definition 4: A Hamilton circuit in a graph G is a simple circuit that passes through every vertex of G exactly once.

Theorem 1: A connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has even degree.

Theorem 2: A connected multigraph with at least two vertices has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree.

Theorem 3 (Dirac's Theorem): If G is a simple graph with $n \geq 3$ vertices such that the degree of every vertex in G is at least $n/2$, then G has a Hamilton circuit.

Theorem 4 (Ore's Theorem): If G is a simple graph with $n \geq 3$ vertices such that $\deg(u) + \deg(v) \geq n$ for every pair of nonadjacent vertices u and v in G, then G has a Hamilton circuit.

Note that Dirac's Theorem is a special case of Ore's Theorem. Both theorems give only sufficient (not necessary) conditions for the existence of Hamilton circuits.