Counting Functions and Relations

Let A and B be finite sets with $|A| = 6$ and $|B| = 9$. Count the number of

1. **relations from A to B.**

   Solution: Each relation from A to B is a subset of $A \times B$. Since there are $|A \times B| = 6 \cdot 9 = 54$ elements in $A \times B$, then there are $2^{54} = 18,014,398,509,481,984$ subsets of $A \times B$ and therefore the same number of relations from A to B.

2. **functions from A to B.**

   Solution: Each of the six elements in A maps to any of the nine elements in B. So by the product rule there are $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 9^6 = 531,441$ such functions.

3. **one-to-one functions from A to B.**

   Solution: The first element of A can map to any of the 9 elements of B. The second element of A can map to any of the remaining 8 elements of B. The third element of A can map to any of the remaining 7 elements of B, and so on. By the product rule there are $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 60,480$ one-to-one functions from A to B.

4. **onto functions from A to B.**

   Solution: There are no onto functions from A to B since a function from A to B can map to at most six but not all nine elements of B.