Proof of Validity of Argument Example 5

Example 5

Everyone has either a brother or a sister.
Everyone either does not have a sister or they are not married.
Anyone who has a child is married.
Someone does not have a brother.

$\therefore$ There is somebody who does not have a child.

Proof of Validity

Let the domain be the set of all people. Define $B(x)$, $S(x)$, $M(x)$ and $C(x)$ to be "x has a brother," "x has a sister," "x is married," and "x has a child," respectively.

The argument has the form

$$\begin{array}{ll} & \forall x(B(x) \vee S(x)) \\ & \forall x(\neg S(x) \vee \neg M(x)) \\ & \forall x(C(x) \to M(x)) \\ & \exists x \neg B(x) \\ \hline \therefore & \exists x \neg C(x) \end{array}$$

Step	Reason
1. $\exists x \neg B(x)$	hypothesis
2. $\neg B(c)$ for some c	existential instantiation (1)
3. $\forall x(B(x) \vee S(x))$	hypothesis
4. $B(c) \vee S(c)$	universal instantiation (3)
5. $S(c)$	disjunctive syllogism (2,4)
6. $\forall x(\neg S(x) \vee \neg M(x))$	hypothesis
7. $\neg S(c) \vee \neg M(c)$	universal instantiation (6)
8. $\neg(\neg S(c))$	double negation (5)
9. $\neg M(c)$	disjunctive syllogism (7,8)
10. $\forall x(C(x) \to M(x))$	hypothesis
11. $C(c) \to M(c)$	universal instantiation (10)
12. $\neg C(c)$	modus tollens (9,11)
13. $\exists x \neg C(x)$	existential generalization (12)