Properties of Sequences

A sequence $\{a_n\}$ **converges** if $\lim_{n\to\infty} a_n = L$ for some real number L, otherwise it **diverges**. Limits of sequences are formally defined in terms of ϵ and δ (see sec 9.1).

Limits of sequences share many of the same properties as limits of functions of a real variable. Below is a list of key properties. In the following L and K refer to real numbers.

1. Let $\{a_n\}$ and $\{b_n\}$ be sequences for which $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = K$. Then

   (a) $\lim_{n\to\infty} (a_n \pm b_n) = L \pm K$

   (b) $\lim_{n\to\infty} (ca_n) = cL$, for any real number c

   (c) $\lim_{n\to\infty} (a_n b_n) = LK$

   (d) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{L}{K}$, if $K \neq 0$ and $b_n \neq 0$

2. **Squeeze Theorem**: Let $\{a_n\}$ and $\{b_n\}$ be sequences for which $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} b_n$. If $\{c_n\}$ is a sequence and there is some fixed integer N for which $a_n \leq c_n \leq b_n$ for all $n > N$, then $\lim_{n\to\infty} c_n = L$.

3. Let $\{a_n\}$ be a sequence. Then $\lim_{n\to\infty} |a_n| = 0$ if, and only if, $\lim_{n\to\infty} a_n = 0$.

4. Let f be a function of a real variable for which $\lim_{x\to\infty} f(x) = L$. If $\{a_n\}$ is a sequence such that $f(n) = a_n$ for every positive integer n, then $\lim_{n\to\infty} a_n = L$.