Mathematics 101 Test #3B **Name:** ______________________________

Instructor: George Ballinger **Section:** ___________

Term: Winter, 2016

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.
Give exact answers (no decimals) unless told otherwise.

Mark: $\overline{25}$

1. Consider the sequence $\{a_n\} = \frac{3}{2}, \frac{4}{4}, \frac{5}{6}, \frac{6}{8}, \frac{7}{10}, \ldots$.

   (a) Find a formula, a_n, for the n^{th} term of the sequence.

[1]

   (b) Determine whether the sequence $\{a_n\}$ converges or diverges. If it converges, find its limit. If it diverges, determine whether it diverges to infinity, negative infinity, or neither.

[1]

   (c) Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges and identify the test you are using.

[1]

   (d) Would the 100th partial sum, $S_{100} = a_1 + a_2 + a_3 + \cdots + a_{100}$, of the series $\sum_{n=1}^{\infty} a_n$ provide a reasonable approximation for the sum of the series? Why or why not?

[1]

2. Find the **exact** sum of each of the following convergent series.

   (a) $\sum_{n=2}^{\infty} \left(\frac{1}{3}\right)^n$

[2]

   (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} 10^n$

[1]

3. Find a geometric power series (using Σ-notation) for $f(x) = \dfrac{1}{2x-5}$ centered at $c = 3$ and determine its interval of convergence.

[4]

4. Verify that the conditions of the integral test are satisfied by the series and use the integral test to determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

[3]

5. Use a comparison test to determine whether the series converges or diverges. Identify which comparison test you are using and show that all of the conditions of the test are satisfied.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+2n+3}$$

[3]

6. Determine whether the series converges conditionally, converges absolutely, or diverges. Identify which tests you are using and show that all of the conditions of the tests are satisfied.

$$\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt[3]{n}}$$

[3]

7. Use the power series $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$ to express $\int_0^{1/2} x \arctan x \, dx$ as a series. Then use the first two nonzero terms of the series to approximate the value of $\int_0^{1/2} x \arctan x \, dx$ and estimate the size of the error by using the Alternating Series Remainder theorem.

[5]