

The Sharp EL-531 calculator may be used on this test.
 Show all of your work in the space provided.
 The number of marks for each question is indicated in brackets.
 Give exact answers (no decimals) unless told otherwise.

Mark:
25

1. Consider the sequence $\{a_n\} = \frac{1(2!)}{3!}, \frac{2(3!)}{4!}, \frac{3(4!)}{5!}, \frac{4(5!)}{6!}, \frac{5(6!)}{7!}, \dots$

(a) Find and **simplify** a formula, a_n , for the n^{th} term of the sequence.

[2]

(b) Determine whether the sequence $\{a_n\}$ converges or diverges. If it converges, find its limit. If it diverges, determine whether it diverges to infinity, negative infinity, or neither.

[1]

(c) Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges and identify the test you are using.

[1]

2. Find the **exact** sum of each of the following convergent series.

(a) $\left(1 - \frac{2}{3}\right) + \left(\frac{2}{3} - \frac{3}{5}\right) + \left(\frac{3}{5} - \frac{4}{7}\right) + \left(\frac{4}{7} - \frac{5}{9}\right) + \dots + \left(\frac{n}{2n-1} - \frac{n+1}{2n+1}\right) + \dots$

[2]

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(\frac{\pi}{2}\right)^{2n+1}$

[1]

3. Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series where $0 < a_n \leq b_n$ for all $n \geq 1$. Circle **all** of the statements that must be true. No justification is required.

[1]

- (a) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges. (d) If $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.
- (b) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges. (e) Either both series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge or they both diverge.
- (c) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

4. Determine whether each series converges or diverges. Identify which test you are using.

(a) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$

[1]

(b) $\sum_{n=0}^{\infty} \frac{1}{\pi^n}$

[1]

5. Consider the series

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}.$$

- (a) Verify that the conditions of the integral test are satisfied by the series and use the integral test to determine whether the series converges or diverges.

[4]

- (b) The 2017th partial sum, S_{2017} , of this series is approximately 2.824156266. Explain whether or not this is a reasonable approximation for the sum of the series.

[1]

6. Find a geometric power series (using Σ -notation) for $f(x) = \frac{5}{2x+3}$ centred at $c = 1$ and determine its interval of convergence.

[4]

7. Use the Maclaurin series for $\cos x$ to express $\int_0^1 \cos \sqrt{x} dx$ as a series. Then use the first three nonzero terms of the series to approximate the value of $\int_0^1 \cos \sqrt{x} dx$ and estimate the size of the error by using the Alternating Series Remainder theorem. Be sure to verify that the conditions of the Alternating Series Remainder theorem are satisfied.

[6]