CAMOSUN

Mathematics 101 Test #2A **Name:** SOLUTIONS

Instructor: George Ballinger **Section:** ____

Term: Winter, 2015

Mark: $\overline{}$ 25

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.
Give exact answers (no decimals) unless told otherwise.

1. Find $\int \frac{-2x+23}{x^2-3x-4}dx$.

$$\frac{-2x+23}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$$

$$-2x+23 = A(x+1) + B(x-4)$$

$$x=4 \Rightarrow 15 = 5A \Rightarrow A=3$$

$$x=-1 \Rightarrow 25 = -5B \Rightarrow B=-5$$

[3]

$$\int \left(\frac{3}{x-4} + \frac{-5}{x+1} \right) dx = 3\ln|x-4| - 5\ln|x+1| + C$$

2. Find $\int \sec^4 x \tan^2 x\, dx$.

$$\int \tan^2 x \sec^2 x \sec^2 x\, dx$$

Let $u = \tan x$

$du = \sec^2 x\, dx$

$$= \int \tan^2 x (\tan^2 x + 1) \sec^2 x\, dx$$

$$= \int (\tan^4 x + \tan^2 x) \sec^2 x\, dx$$

[3]

$$= \frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C$$

3. Find $\int 8x \arctan 2x \, dx$.

Let $u = \arctan 2x \qquad dv = 8x\,dx$

$du = \frac{2}{1+4x^2}dx \qquad v = 4x^2$

$$\begin{array}{r|l} & 2 \\ \hline 4x^2+1 & 8x^2 \\ & 8x^2+2 \\ \hline & -2 \end{array}$$

$$\int 8x \arctan 2x\,dx = 4x^2 \arctan 2x - \int \frac{8x^2}{1+4x^2}dx$$

$$= 4x^2 \arctan 2x - \int \left(2 - \frac{2}{1+4x^2}\right)dx$$

[5] $$= 4x^2 \arctan 2x - \left[2x - \frac{2}{2}\arctan 2x\right] + C$$

$$= 4x^2 \arctan 2x - 2x + \arctan 2x + C$$

4.

(a) Find $\int \frac{1}{x^2(1+e^{1/x})}dx$ by using the integration formula $\int \frac{1}{1+e^u}du = u - \ln(1+e^u) + C$.

Let $u = \frac{1}{x} \qquad du = \frac{-1}{x^2}dx$

$$\int \frac{1}{x^2(1+e^{1/x})}dx = -\int \frac{1}{1+e^{1/x}}\left(-\frac{1}{x^2}\right)dx = -\int \frac{1}{1+e^u}du$$

[3] $$= -u + \ln(1+e^u) + C = -\frac{1}{x} + \ln\left(1+e^{\frac{1}{x}}\right) + C$$

(b) Use your answer from part (a) to evaluate the **improper** integral, or if it diverges, then determine whether it diverges to infinity, to negative infinity, or neither. Be sure to convert the integral to a limit.

$$\int_1^\infty \frac{1}{x^2(1+e^{1/x})}dx = \lim_{b\to\infty} \int_1^b \frac{1}{x^2(1+e^{1/x})}dx = \lim_{b\to\infty}\left[-\frac{1}{x} + \ln\left(1+e^{\frac{1}{x}}\right)\right]_1^b$$

[2] $$= \lim_{b\to\infty}\left[\left(-\frac{1}{b} + \ln\left(1+e^{\frac{1}{b}}\right)\right) - \left(-1 + \ln(1+e)\right)\right]$$

$$= \left(0 + \ln(1+1)\right) - \left(-1 + \ln(1+e)\right)$$

$$= \ln 2 + 1 - \ln(1+e)$$

5. Evaluate each limit, or if it does not exist, then determine whether it is infinity, negative infinity or neither.

(a) $\lim_{x\to\infty} \frac{\ln x}{\ln(x^2+1)} \overset{L'HR}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{2x}{x^2+1}} = \lim_{x\to\infty} \frac{x^2+1}{2x^2} = \lim_{x\to\infty}\left(\frac{1}{2}+\frac{1}{2x^2}\right) = \frac{1}{2}$

$\frac{\infty}{\infty}$

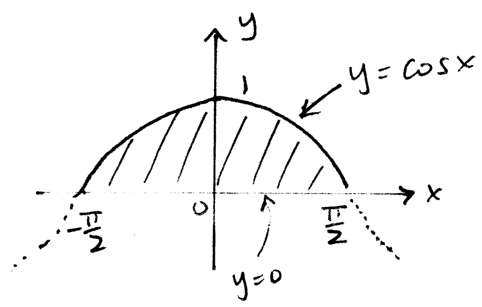
[2]

(b) $\lim_{x\to 0^+} \frac{\ln x}{\ln(x^2+1)} = -\infty$ (not indeterminate)

$\frac{-\infty}{0^+}$

[1]

6. Find the y-coordinate, $\bar{y}$, of the centroid of the planer region bounded by the curves $y=\cos x$ and $y=0$ for $-\pi/2 \le x \le \pi/2$. Include a sketch of the curves and shade the region.



$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = 2\int_0^{\frac{\pi}{2}} \cos x\,dx$ (by symmetry)

$= \left[2\sin x\right]_0^{\frac{\pi}{2}} = 2 - 0 = 2$

[6] $A_x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{2}\cdot \cos x\,dx = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 x\,dx$

$= \int_0^{\frac{\pi}{2}} \cos^2 x\,dx$ (by symmetry)

$= \int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2}\,dx = \left[\frac{1}{2}x + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$

$\therefore \bar{y} = \frac{A_x}{A} = \frac{\pi/4}{2} = \frac{\pi}{8}$