Mathematics 101 Test #2A **Name:** SOLUTIONS

Instructor: George Ballinger **Section:** ____________

Term: Winter, 2016

> The Sharp EL-531 calculator may be used on this test.
> Show all of your work in the space provided.
> The number of marks for each question is indicated in brackets.
> Give exact answers (no decimals) unless told otherwise.

Mark: $\overline{}$ 25

1. Integrate $\int \frac{1+\sin\theta}{\cos^2\theta}\,d\theta$.

$$= \int\left(\frac{1}{\cos^2\theta} + \frac{\sin\theta}{\cos^2\theta}\right)d\theta = \int(\sec^2\theta + \sec\theta\tan\theta)\,d\theta = \tan\theta + \sec\theta + C$$

[3]

2. Find $\int\sqrt{e^{2x}-1}\,dx$ by using the integration formula $\int\frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - \operatorname{arcsec}\frac{|u|}{a} + C$ for $a>0$.

Let $u = e^x$ and $a = 1$

$du = e^x dx$

[3]

$\therefore dx = \frac{du}{e^x} = \frac{du}{u}$

$$\int\sqrt{e^{2x}-1}\,dx = \int\sqrt{u^2-1}\cdot\frac{1}{u}\,du$$

$$= \sqrt{u^2-1} - \operatorname{arcsec}|u| + C$$

$$= \sqrt{e^{2x}-1} - \operatorname{arcsec}(e^x) + C$$

3. Use partial fractions to integrate $\int\frac{3x+1}{(x-1)(x^2+1)}\,dx$.

$$\frac{3x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$3x+1 = A(x^2+1) + (Bx+C)(x-1)$$

[5]

$x=1 \Rightarrow 4 = 2A \Rightarrow A = 2$

$x=0 \rightarrow 1 = A - C \Rightarrow C = A-1 = 2-1 = 1$

$x=2 \Rightarrow 7 = 5A+2B+C \Rightarrow B = \frac{1}{2}[7-5A-C] = \frac{1}{2}[7-10-1] = -2$

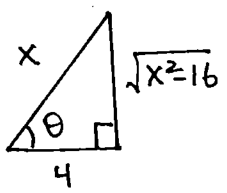
$$\int\left[\frac{2}{x-1} + \frac{-2x+1}{x^2+1}\right]dx = \int\left[\frac{2}{x-1} - \frac{2x}{x^2+1} + \frac{1}{x^2+1}\right]dx$$

$$= 2\ln|x-1| - \ln|x^2+1| + \arctan x + C$$

4. Use a trigonometric substitution to integrate $\int \frac{\sqrt{x^2-16}}{x}\,dx$.

Let $x = 4\sec\theta \Rightarrow \sec\theta = \frac{x}{4}$

$dx = 4\sec\theta\tan\theta\,d\theta$



$\tan\theta = \frac{\sqrt{x^2-16}}{4}$

[5] $\sqrt{x^2-16} = 4\tan\theta$

$$\int \frac{\sqrt{x^2-16}}{x}\,dx = \int \frac{4\tan\theta}{4\sec\theta}\cdot 4\sec\theta\tan\theta\,d\theta$$

$$= 4\int \tan^2\theta\,d\theta$$

$$= 4\int (\sec^2\theta - 1)\,d\theta$$

$$= 4(\tan\theta - \theta) + C$$

$$= 4\tan\theta - 4\theta + C$$

$$= \sqrt{x^2-16} - \underbrace{4\operatorname{arcsec}\left(\frac{x}{4}\right)}_{\text{OR } \arctan\left(\frac{\sqrt{x^2-16}}{4}\right) \text{ etc.}} + C$$

5. Evaluate the **improper** integral, or if it diverges, then determine whether it diverges to infinity, to negative infinity, or neither. Be sure to convert the integral to limits.

$$\int_{-1}^{1} \frac{1}{x^2}\,dx = \int_{-1}^{0} \frac{1}{x^2}\,dx + \int_{0}^{1} \frac{1}{x^2}\,dx = \lim_{b\to 0^-} \int_{-1}^{b} x^{-2}\,dx + \lim_{a\to 0^+} \int_{a}^{1} x^{-2}\,dx$$

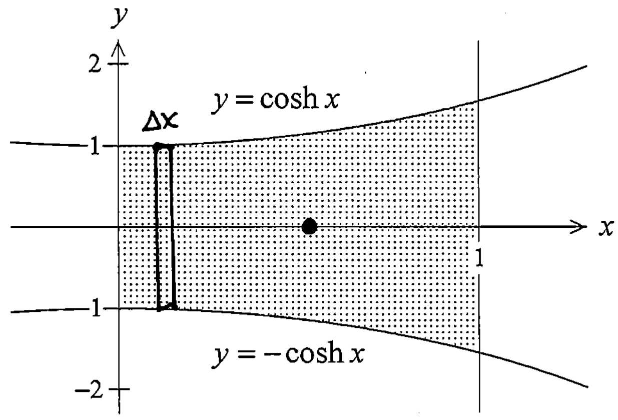
$$= \lim_{b\to 0^-} \left[-\frac{1}{x}\right]_{-1}^{b} + \lim_{a\to 0^+} \left[-\frac{1}{x}\right]_{a}^{1}$$

[4] $$= \underbrace{\lim_{b\to 0^-} \left[-\frac{1}{b} - 1\right]}_{\infty} + \underbrace{\lim_{a\to 0^+} \left[-1 + \frac{1}{a}\right]}_{\infty} = \infty$$

$$= \infty + \infty$$

Integral diverges to ∞

6. Find the centroid of the region bounded by the curves $y=\cosh x$, $y=-\cosh x$, $x=0$ and $x=1$. Round your answer to four decimal places.



for $0 \le x \le 1$

$$\Delta A = (\cosh x - (-\cosh x))\,\Delta x$$
$$= 2\cosh x \,\Delta x$$

$$A = 2\int_0^1 \cosh x\,dx$$
$$= 2\sinh x \Big|_0^1 = 2\sinh 1 \approx 2.3504$$

$$A_x = 0 \Rightarrow \bar{y} = 0 \quad \text{(symmetry)}$$

$$\Delta A_y = x\,(2\cosh x)\,\Delta x$$

[5] $$A_y = \int_0^1 \underbrace{2x}_{u}\, \underbrace{\cosh x\,dx}_{dv}$$

	u	v'
+	$2x$	$\cosh x$
−	2	$\sinh x$
+	0	$\cosh x$

$$= \Big[2x\sinh x - 2\cosh x\Big]_0^1$$
$$= 2\sinh 1 - 2\cosh 1 + 2 \approx 1.2642$$

$$\bar{x} = \frac{A_y}{A} = \frac{2\sinh 1 - 2\cosh 1 + 2}{2\sinh 1} = 1 - \coth 1 + \operatorname{csch} 1 \approx 0.5379$$

$\therefore$ Centroid is $(\bar{x},\bar{y}) = (0.5379, 0)$