CAMOSUN

Mathematics 101 Test #2B Name: SOLUTIONS

Instructor: George Ballinger Section: ___________

Term: Winter, 2017

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.
Give exact answers (no decimals) unless told otherwise.

Mark: $\frac{\quad}{25}$

1. Integrate $\int \sin^8\theta \cos^3\theta \, d\theta$.

$$= \int \sin^8\theta \cos^2\theta \cos\theta \, d\theta$$

$$= \int \sin^8\theta \left(1 - \sin^2\theta\right) \cos\theta \, d\theta$$

[3]

$$= \int \left(\sin^8\theta - \sin^{10}\theta\right) \cos\theta \, d\theta$$

$$= \frac{1}{9}\sin^9\theta - \frac{1}{11}\sin^{11}\theta + C$$

2. Evaluate $\int_1^e \ln x \, dx$.

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

$$\int \ln x \, dx = x\ln x - \int dx = x\ln x - x$$

[3]

$$\therefore \int_1^e \ln x \, dx = \left[x\ln x - x\right]_1^e = (e-e) - (0-1) = 1$$

3. Evaluate the limit, or if it does not exist, then determine whether it is infinity, negative infinity, or neither.

$$\lim_{x\to 0}\frac{\arctan x}{\tan x} \overset{L'HR}{=} \lim_{x\to 0}\frac{\frac{1}{1+x^2}}{\sec^2 x} = \frac{1}{1} = 1$$

form $\frac{0}{0}$

[2]

4. Evaluate the improper integral, or if it does not exist, then determine whether it is infinity, negative infinity, or neither. Be sure to convert the integral to a limit.

$$\int_{10}^{\infty}\frac{5}{x^2}\,dx = \lim_{b\to\infty}\int_{10}^{b}\frac{5}{x^2}\,dx = \lim_{b\to\infty}\left[-\frac{5}{x}\right]_{10}^{b} = \lim_{b\to\infty}\left[\overset{\to 0}{\cancel{-\frac{5}{b}}} + \frac{1}{2}\right] = \frac{1}{2}$$

[2]

5. Use partial fractions to integrate $\int \frac{5x^2+x+12}{x(x^2+4)}dx$.

$$\frac{5x^2+x+12}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$5x^2+x+12 = A(x^2+4) + (Bx+c)x = Ax^2 + 4A + Bx^2 + Cx$$

Coeff. of x^2 : $5 = A+B$

x : $1 = C$

1 : $12 = 4A \Rightarrow A = 3$ $\longrightarrow$ $\therefore B = 2$

[5]

$$\int\left(\frac{3}{x} + \frac{2x+1}{x^2+4}\right)dx = \int \frac{3}{x}dx + \int \frac{2x}{x^2+4}dx + \int \frac{1}{x^2+4}dx$$

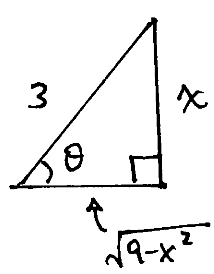
$$= 3\ln|x| + \ln|x^2+4| + \frac{1}{2}\arctan\frac{x}{2} + C$$

6. Use a trigonometric substitution to integrate $\int \frac{\sqrt{9-x^2}}{x}dx$.

Let $x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$\sin\theta = \frac{x}{3}$



[5]

$\sqrt{9-x^2} = 3\cos\theta$

$$= \int \frac{3\cos\theta}{3\sin\theta} \cdot 3\cos\theta\, d\theta$$

$$= \int \frac{3\cos^2\theta}{\sin\theta}d\theta = 3\int \frac{1-\sin^2\theta}{\sin\theta}d\theta$$

$$= 3\int(\csc\theta - \sin\theta)\,d\theta$$

$$= 3\left[-\ln|\csc\theta+\cot\theta| + \cos\theta\right] + C$$

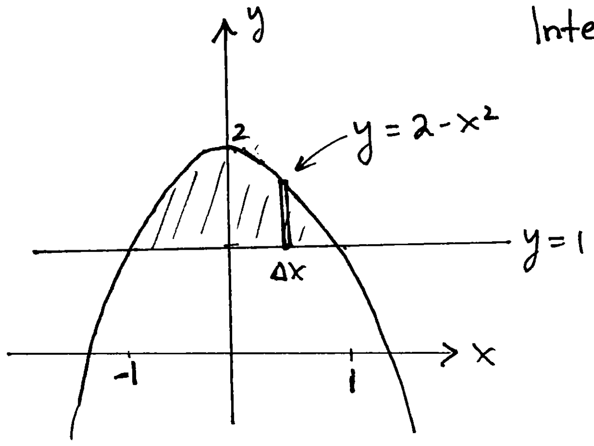
$$= -3\ln\left|\frac{3}{x} + \frac{\sqrt{9-x^2}}{x}\right| + \sqrt{9-x^2} + C$$

$$= -3\ln\left|\frac{3+\sqrt{9-x^2}}{x}\right| + \sqrt{9-x^2} + C$$

7. Sketch and shade the region in the plane bounded by the curves

$$y = 2 - x^2 \text{ and } y = 1,$$

and then find the centroid $(\bar{x}, \bar{y})$ of the region. Note the symmetry of the region and use it to simplify your calculations.



Intersection: $2 - x^2 = 1$

$x^2 = 1$

$x = \pm 1$

$$\Delta A = (2 - x^2 - 1)\Delta x = (1 - x^2)\Delta x \quad \text{for } -1 \le x \le 1$$

$$A = \int_{-1}^{1} (1 - x^2)\,dx = 2\int_0^1 (1 - x^2)\,dx = 2\left[x - \tfrac{1}{3}x^3\right]_0^1 = \frac{4}{3}$$

[5] $A_y = 0 \Rightarrow \bar{x} = 0$ by symmetry

$$\Delta A_x = \frac{(2 - x^2) + 1}{2} \cdot \left[(2 - x^2) - 1\right]\Delta x = \frac{(3 - x^2)(1 - x^2)}{2}\,\Delta x \quad \text{for } -1 \le x \le 1$$

$$A_x = \frac{1}{2}\int_{-1}^{1} (3 - x^2)(1 - x^2)\,dx = \int_0^1 (3 - x^2)(1 - x^2)\,dx$$

$$= \int_0^1 (3 - 4x^2 + x^4)\,dx = \left[3x - \tfrac{4}{3}x^3 + \tfrac{1}{5}x^5\right]_0^1 = \frac{28}{15}$$

$$\therefore \bar{y} = \frac{A_x}{A} = \frac{28/15}{4/3} = 7/5$$

$$(\bar{x}, \bar{y}) = (0, 7/5)$$