CAMOSUN

**Mathematics 101 Test #2A** **Name:** SOLUTIONS

**Instructor: George Ballinger** **Section:** ____________

**Term: Winter, 2018**

**Mark:** $\frac{\quad}{25}$

| |
|---|
| The Sharp EL-531 calculator may be used on this test. |
| Show all of your work in the space provided. |
| The number of marks for each question is indicated in brackets. |
| Give exact answers (no decimals) unless told otherwise. |

1. Use partial fractions to integrate $\int \frac{3x+4}{x^2+2x}\,dx$.

$$\frac{3x+4}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

$$3x+4 = A(x+2) + Bx$$

[3] $x=0 \Rightarrow 4 = 2A \Rightarrow A = 2$

$$x=-2 \Rightarrow -2 = -2B \Rightarrow B = 1$$

$$\int\left(\frac{2}{x} + \frac{1}{x+2}\right)dx = 2\ln|x| + \ln|x+2| + C$$

2. Find $\int \tan^3 x \sec^5 x\,dx$.

$$= \int \tan^2 x \sec^4 x \sec x \tan x\,dx$$

$$= \int (\sec^2 x - 1)\sec^4 x \sec x \tan x\,dx$$

[3] $= \int (\sec^6 x - \sec^4 x)\underbrace{\sec x \tan x\,dx}_{du,\text{ where } u=\sec x}$

$$= \frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$$

3. Evaluate $\lim_{x\to\infty} x^{\frac{1}{\ln x}}$. Form $\infty^0$.

Let $L = \lim_{x\to\infty} x^{\frac{1}{\ln x}}$. Then $\ln L = \ln\left(\lim_{x\to\infty} x^{\frac{1}{\ln x}}\right) = \lim_{x\to\infty} \ln\left(x^{\frac{1}{\ln x}}\right)$

[3] $= \lim_{x\to\infty} \frac{1}{\ln x}\cdot \ln x = \lim_{x\to\infty} 1 = 1$

$$\therefore L = e^1 = e$$

4. Integrate by parts by letting $u = \sin x - x\cos x$ and simplify your answer.

$$I = \int \frac{\sin x - x\cos x}{x^2}\,dx$$

$$dv = \frac{1}{x^2}\,dx \qquad v = \frac{-1}{x}$$

Let $u = \sin x - x\cos x$

$du = (\cos x + x\sin x - \cos x)\,dx = x\sin x\,dx$

$$I = (\sin x - x\cos x)\left(-\frac{1}{x}\right) - \int \left(-\frac{1}{x}\right)(x\sin x)\,dx$$

[3] $$= -\frac{\sin x}{x} + \cos x + \int \sin x\,dx$$

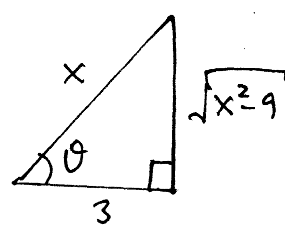
$$= -\frac{\sin x}{x} + \cos x - \cos x + C$$

$$= -\frac{\sin x}{x} + C$$

5. Use a trigonometric substitution to integrate $\int \frac{1}{x(x^2-9)^{3/2}}\,dx$.

Let $x = 3\sec\theta \implies \sec\theta = \frac{x}{3}$ $\qquad \theta = \operatorname{arcsec}\frac{x}{3}$

$dx = 3\sec\theta\tan\theta\,d\theta$




$\tan\theta = \frac{\sqrt{x^2-9}}{3}$

$\implies \sqrt{x^2-9} = 3\tan\theta$

[5] $$\int \frac{1}{x(x^2-9)^{3/2}}\,dx = \int \frac{3\sec\theta\tan\theta\,d\theta}{(3\sec\theta)(3\tan\theta)^3} = \frac{1}{27}\int \cot^2\theta\,d\theta$$

$$= \frac{1}{27}\int(\csc^2\theta - 1)\,d\theta = \frac{1}{27}\left(-\cot\theta - \theta\right) + C$$

$$= -\frac{1}{27}\left(\frac{3}{\sqrt{x^2-9}} + \operatorname{arcsec}\frac{x}{3}\right) + C$$

$$= \frac{-1}{9\sqrt{x^2-9}} - \frac{1}{27}\operatorname{arcsec}\frac{x}{3} + C$$

6. Evaluate the improper integral, or if it does not exist, then determine whether it is infinity, negative infinity, or neither. Be sure to convert the integral to a one-sided limit.

$$\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx = \lim_{b\to 2^-} \int_0^b \frac{1}{\sqrt{4-x^2}}\,dx = \lim_{b\to 2^-} \arcsin\frac{x}{2}\Big|_0^b$$

$$= \lim_{b\to 2^-}\left(\arcsin\frac{b}{2} - \arcsin 0\right)$$

$$= \arcsin 1 - \arcsin 0$$

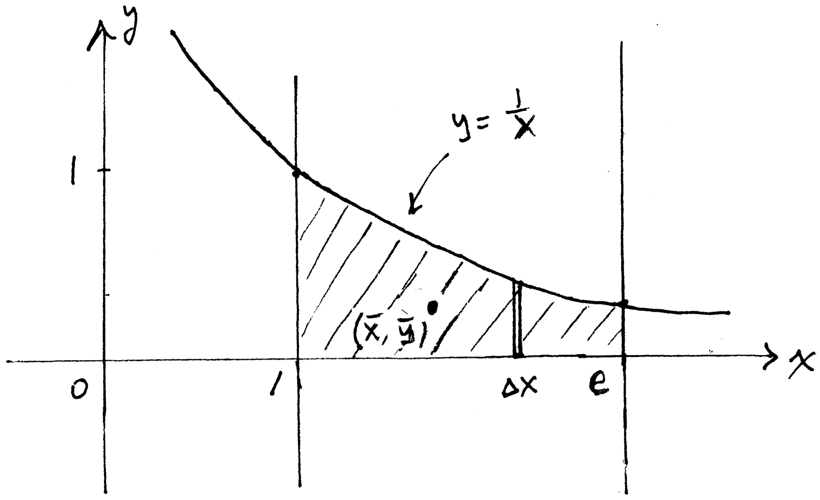
[3]

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

7. Sketch and shade the region in the plane bounded by the curves

$$y = \frac{1}{x},\ y = 0,\ x = 1, \text{ and } x = e,$$

and then compute the centroid $(\bar{x}, \bar{y})$ of the region and plot the point on your graph.




[5]

for $1 \le x \le e$

$$\Delta A = \frac{1}{x}\Delta x \Rightarrow A = \int_1^e \frac{1}{x}\,dx = \ln|x|\Big|_1^e = \ln e - \ln 1 = 1$$

$$\Delta A_y = x \cdot \frac{1}{x}\Delta x \Rightarrow A_y = \int_1^e 1\,dx = x\Big|_1^e = e - 1$$

$$\Delta A_x = \frac{1}{2}\cdot\frac{1}{x}\cdot\frac{1}{x}\Delta x \Rightarrow A_x = \frac{1}{2}\int_1^e \frac{1}{x^2}\,dx = -\frac{1}{2x}\Big|_1^e = -\frac{1}{2e} + \frac{1}{2}$$

$$\therefore (\bar{x}, \bar{y}) = \left(\frac{A_y}{A}, \frac{A_x}{A}\right) = \left(e-1, -\frac{1}{2e} + \frac{1}{2}\right) \approx (1.718, 0.316)$$