Mark: $\overline{\mathbf{25}}$

MATH 101 (Winter, 2023)
Test 2B

1. (4 marks) Evaluate the limit $\displaystyle\lim_{x\to1}\frac{2x-2-\ln x^2}{1+\cos\pi x}$.

2. (4 marks) Write the improper integral in the form of a limit and then evaluate it, or if it diverges, then determine whether it diverges to ∞, $-\infty$, or neither.

$$\int_1^\infty \frac{e^{1/x}}{x^2}\,dx$$

3. Find the following integrals.

    (a) (3 marks) $\displaystyle\int x^5 \ln x \, dx$

    (b) (4 marks) $\displaystyle\int \frac{x^2 - 12}{x(x^2 + 4)} \, dx$

4. (5 marks) Use a trigonometric substitution to evaluate the integral $\displaystyle\int_0^2 \frac{x^2}{\sqrt{16-x^2}}\,dx$.

5. (5 marks) Each end of a swimming pool is in the shape of the semicircle $x^2 + y^2 = 25$ for $y \leq 0$, where x and y are measured in feet, as illustrated. If the pool is full of water weighing 62.4 lb/ft^3, then how much force does the water exert on the wall at each end of the pool?