CAMOSUN

Mathematics 101 Test #1A Name: SOLUTIONS

Instructor: George Ballinger Section: ____

Term: Winter, 2015

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.
Give exact answers (no decimals) unless told otherwise.

Mark: $\overline{}\,25$

1. Fill in the blanks with meaningful answers. No justification is required.

   (a) The domain of $f(x) = \arccos x$ is $[-1, 1]$.

   (b) The range of $f(x) = \arccos x$ is $[0, \pi]$.

   (c) $\arccos(\cos 2\pi) = 0$.

   (d) $\frac{d}{dx}(\arcsin x + \arccos x) = 0$.

[4]

   (e) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$.

   (f) In terms of exponential functions, the hyperbolic sine function is defined by $\sinh x = \frac{e^x - e^{-x}}{2}$.

   (g) $\cosh^{-1} 2 \approx 1.317$. (Approximate using your calculator and round to 3 decimal places.)

   (h) $\frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2}$.

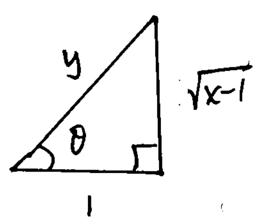
2. Write the expression $\sec(\arctan\sqrt{x-1})$ in algebraic form.

Let $\theta = \arctan\sqrt{x-1}$

$\tan\theta = \sqrt{x-1}$

$\sec(\arctan\sqrt{x-1}) = \sec\theta = y = \sqrt{x}$

[2]



$$y = \sqrt{1^2 + (\sqrt{x-1})^2}$$
$$= \sqrt{1 + x - 1}$$
$$= \sqrt{x}$$

3. Find and simplify the derivative of $y = 3x\arcsin 3x + \sqrt{1-9x^2}$.

$$y' = 3x \cdot \frac{3}{\sqrt{1-9x^2}} + 3\arcsin 3x + \frac{-18x}{2\sqrt{1-9x^2}}$$

[3]

$$= \frac{9x}{\sqrt{1-9x^2}} + 3\arcsin 3x - \frac{9x}{\sqrt{1-9x^2}}$$

$$= 3\arcsin 3x$$

4. Evaluate the integral $\int_{-2}^{-1} \frac{6}{\sqrt{-x^2-4x}}\,dx$.

$$= \int_{-2}^{-1} \frac{6}{\sqrt{4-(x^2+4x+4)}}\,dx = \int_{-2}^{-1} \frac{6}{\sqrt{2^2-(x+2)^2}}\,dx$$

$$= 6\arcsin\frac{x+2}{2}\Big|_{-2}^{-1} = 6\arcsin\frac{1}{2} - 6\arcsin 0$$

[3] $$= 6\cdot\frac{\pi}{6} - 6\cdot 0 = \pi$$

5. Find the area of the region in the plane bounded by the curves $y = x^2 - 4x$ and $y = -x$. Include a sketch of the curves and shade the region.

Intersection: $x^2-4x=-x$

$x^2-3x=0$

$x(x-3)=0$

$x=0$ or $x=3$

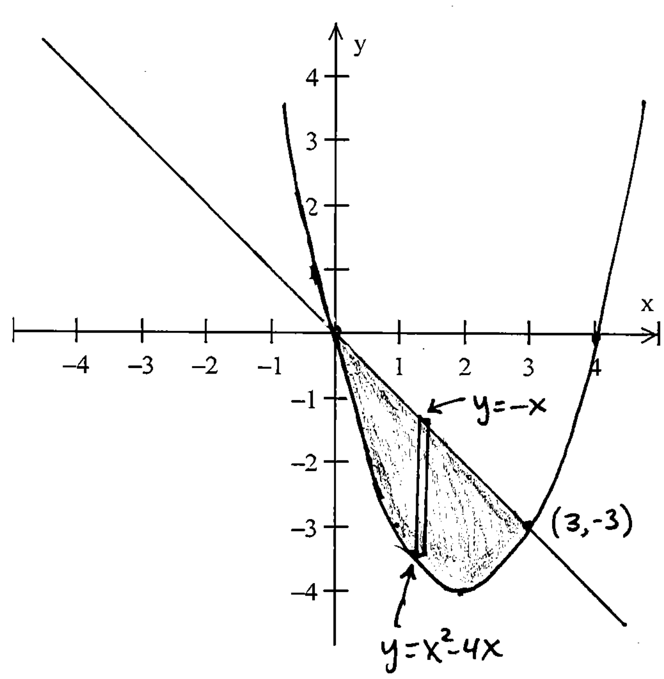
$\Delta A = (-x-(x^2-4x))\,\Delta x$ for $0 \le x \le 3$

$\therefore A = \int_0^3 (-x^2+3x)\,dx$

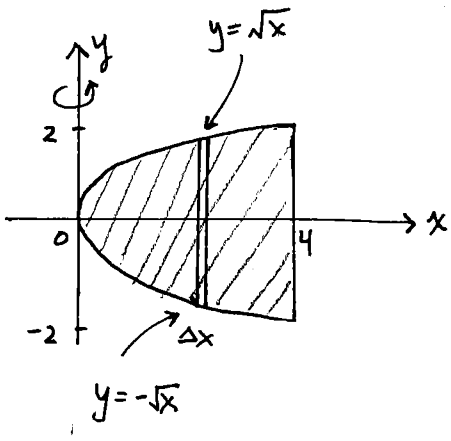
$= \left[-\frac{1}{3}x^3+\frac{3}{2}x^2\right]_0^3$

[3] $= -9+\frac{27}{2}$

$= \frac{9}{2}$



6. Using the **cylindrical shell** method, find the volume of the solid formed by revolving the region in the plane bounded by the curves $x = y^2$ and $x = 4$ about the y-**axis**.



$\Delta V = 2\pi p(x)\, h(x)\, \Delta x$ for $0 \le x \le 4$

where $p(x) = x$ and $h(x) = \sqrt{x} - (-\sqrt{x}) = 2\sqrt{x}$

$$\therefore V = 2\pi \int_0^4 x \cdot 2\sqrt{x}\, dx$$

$$= 4\pi \int_0^4 x^{3/2}\, dx$$

$$= 4\pi \cdot \frac{2}{5} x^{5/2} \Big|_0^4$$

$$= \frac{8\pi}{5} \cdot 32$$

$$= \frac{256\pi}{5}$$

[4]

7. Find the arc length of the catenary given by $y = \frac{1}{3}\cosh 3x$ on the interval $-1 \le x \le 1$.

$y' = \frac{1}{3}\sinh 3x \cdot 3 = \sinh 3x$

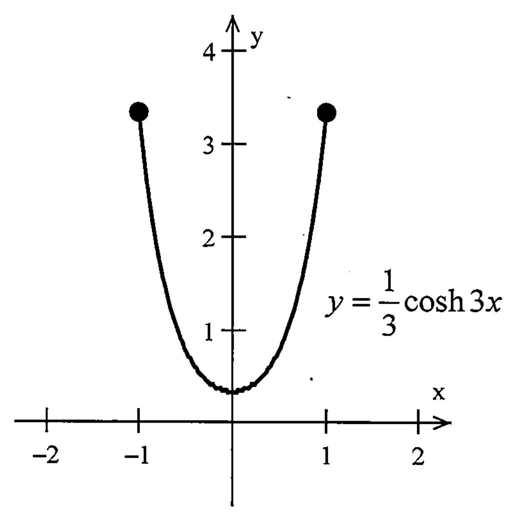
$ds = \sqrt{1 + \sinh^2 3x}\, dx = \cosh 3x\, dx$

$$S = \int_{-1}^{1} \cosh 3x\, dx = 2\int_0^1 \cosh 3x\, dx$$

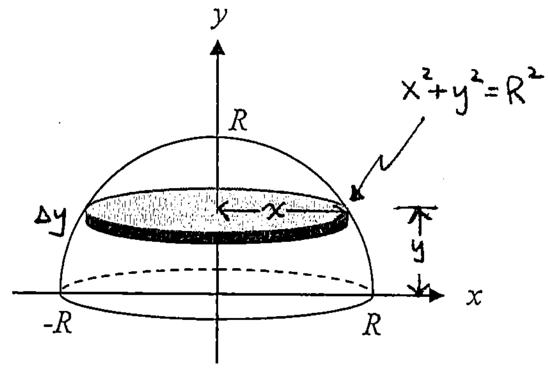
by symmetry

$$= \frac{2}{3}\sinh 3x \Big|_0^1 = \frac{2}{3}\sinh 3$$

[3]



8. A hemispherical "dome-shaped" tank having radius R is sitting flat on the ground as shown in the figure. Find the total amount of work required to completely fill the tank through a hole in the bottom of the tank with a fluid having weight density ρg.



$\Delta W = \Delta F \cdot d$ for $0 \le y \le R$

where $d = y$ and $\Delta F = \rho g \cdot \pi x^2 \Delta y$

where $x^2 = R^2 - y^2$

$\therefore \Delta W = \rho g \pi (R^2 - y^2) y \Delta y$

[3] $W = \rho g \pi \int_0^R (R^2 y - y^3)\, dy$

$= \rho g \pi \left[\frac{1}{2}R^2 y^2 - \frac{1}{4}y^4\right]_0^R$

$= \rho g \pi \left[\frac{1}{2}R^4 - \frac{1}{4}R^4\right]$

$= \frac{\rho g \pi R^4}{4}$