CAMOSUN

Mathematics 101 Test #1B **Name:** SOLUTIONS

Instructor: George Ballinger **Section:** ____________

Term: Winter, 2016

Mark: $\frac{\quad}{25}$

> The Sharp EL-531 calculator may be used on this test.
> Show all of your work in the space provided.
> The number of marks for each question is indicated in brackets.
> Give exact answers (no decimals) unless told otherwise.

1. Evaluate $\arctan 0.7$ and $\tanh^{-1} 0.7$ using your calculator, rounding each to three decimal places.

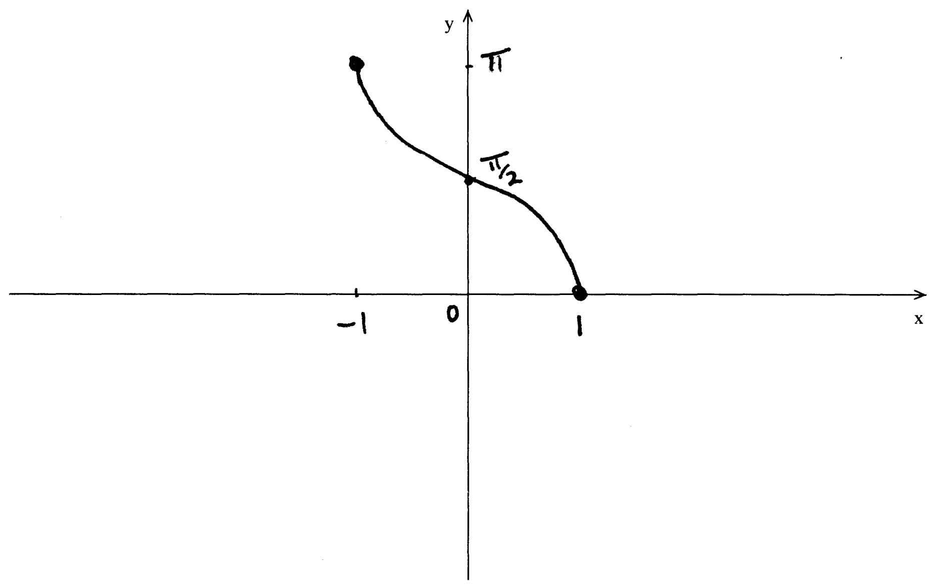
[1]

$\arctan 0.7 \approx 0.611$ (RAD!)

$\tanh^{-1} 0.7 \approx 0.867$

2. Sketch the graph of $y = \arccos x$, clearly showing its domain, range, intercepts and $\underbrace{\text{asymptotes}}_{\text{none}}$ (if any).

[2]



3. Simplify: $\ln(\cosh x - \sinh x)$

$$\ln\left(\frac{e^x+e^{-x}}{2} - \frac{e^x-e^{-x}}{2}\right) = \ln\left(e^{-x}\right) = -x$$

[2]

4. Differentiate and simplify your answer: $y = \arcsin(\cos x)$, where $0 < x < \pi$

$$y' = \frac{1}{\sqrt{1-\cos^2 x}} \cdot (-\sin x) = \frac{-\sin x}{\sqrt{\sin^2 x}}$$

[2]

$$= -\frac{\sin x}{|\sin x|} = \frac{-\sin x}{\sin x} = -1$$

(since $\sin x > 0$ for $0 < x < \pi$)

5. Integrate and simplify your answer: $\displaystyle\int \frac{5\sinh x}{\sqrt{\sinh^2 x+1}}\,dx$

$$= 5\int \frac{\sinh x}{\sqrt{\cosh^2 x}}\,dx = 5\underbrace{\int \frac{\sinh x}{\cosh x}\,dx}_{\text{form } \int \frac{du}{u}} = 5\ln|\cosh x| + C$$

or $5\ln(\cosh x) + C$

[3]

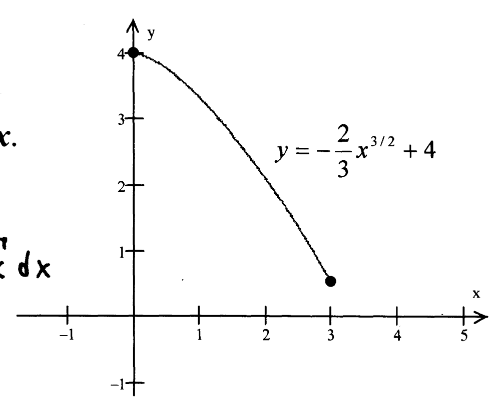
6. Evaluate: $\displaystyle\int_{-4}^{-1} \frac{1}{x^2+8x+25}\,dx$

$$\int_{-4}^{-1} \frac{1}{x^2+8x+16+9}\,dx = \int_{-4}^{-1} \frac{1}{3^2+(x+4)^2}\,dx = \frac{1}{3}\arctan\left(\frac{x+4}{3}\right)\Big|_{-4}^{-1}$$

$$= \frac{1}{3}\arctan 1 - \frac{1}{3}\arctan 0 = \frac{1}{3}\cdot\frac{\pi}{4} - \frac{1}{3}\cdot 0 = \frac{\pi}{12}$$

[3]

7. Consider the curve shown to the right, where $0 \le x \le 3$.



(a) Find the arc length of the curve by integrating **with respect to *x***.

$y' = -x^{1/2}$

$ds = \sqrt{1+(y')^2}\,dx = \sqrt{1+(-x^{1/2})^2}\,dx = \sqrt{1+x}\,dx$

[3] $\displaystyle S = \int_0^3 (1+x)^{1/2}\,dx = \frac{2}{3}(1+x)^{3/2}\Big|_0^3$

$$= \frac{2}{3}(8-1) = \frac{14}{3}$$

(b) Set up **but do not evaluate** a definite integral **with respect to *x***, representing the surface area of the surface formed by revolving the curve shown above about the *y*-axis.

[1] $\Delta S = 2\pi r\,ds$ where $r = x$ and $ds = \sqrt{1+x}\,dx$ as above

$$\therefore\ S = 2\pi\int_0^3 x\sqrt{1+x}\,dx$$

(ANS: $\frac{232}{15}\pi$)

8. Consider the region in the plane bounded by the curves

$$y = \sqrt{x} \text{ and } y = x^2.$$

Set up **but do not evaluate** a definite integral representing the volume of the solid formed by revolving the region about the line $y = 1$ using the **shell** method.

Intersection

$x^2 = \sqrt{x}$

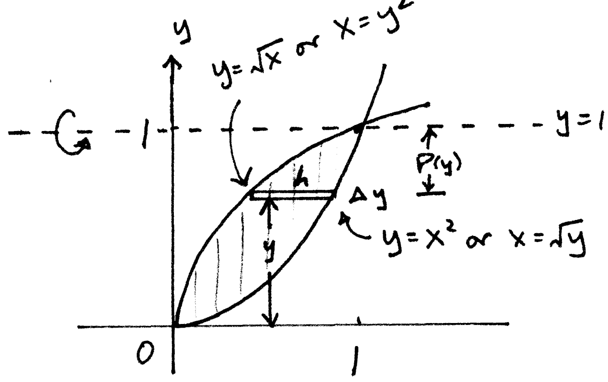
$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0, \; x = 1$

$y = 0, \; y = 1$



[4]

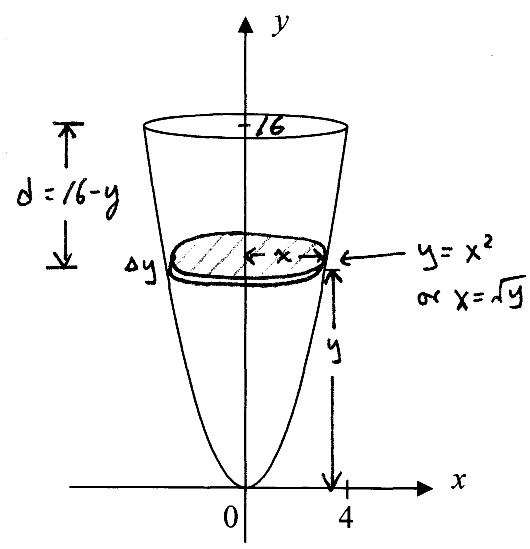
for $0 \le y \le 1$, $\Delta V = 2\pi \, p(y) \, h(y) \, \Delta y$

where $p(y) = 1 - y$ and $h(y) = \sqrt{y} - y^2$

$$\therefore \; V = 2\pi \int_0^1 (1-y)(\sqrt{y} - y^2)\, dy$$

(ANS: $\frac{11}{30}\pi$)

9. Consider a tank whose surface is formed by revolving the curve $y = x^2$, for $0 \le x \le 4$, about the y-axis (where x and y are measured in meters). If the tank is full of water having weight density ρg = 9,800 N/m^3, then how much work is required to pump all the water out over the top?



$\Delta W = \Delta F \cdot d$ for $0 \le y \le 16$

where $d = 16 - y$ and

$\Delta F = \rho g \, \Delta V = \rho g \, \pi x^2 \Delta y = \rho g \pi y \, \Delta y$

$$\therefore \; W = \rho g \pi \int_0^{16} y(16-y)\, dy$$

[4]

$$= \rho g \pi \int_0^{16} (16y - y^2)\, dy$$

$$= \rho g \pi \left[8y^2 - \tfrac{1}{3} y^3 \right]_0^{16}$$

$$= \rho g \pi \left(\frac{2048}{3} \right)$$

$$= (9800\pi)\left(\frac{2048}{3} \right)$$

$$= \frac{20\,070\,400\,\pi}{3} \text{ N}\cdot\text{m}$$

$$\approx 2.10 \times 10^7 \text{ N}\cdot\text{m}$$