CAMOSUN

Mathematics 101 Test #1A Name: SOLUTIONS

Instructor: George Ballinger **Section:** ____________

Term: Winter, 2017

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.
Give exact answers (no decimals) unless told otherwise.

Mark: $\overline{\quad}$ 25

1. Evaluate $\coth 0.6$ and $\tanh^{-1} 0.6$ using your calculator, rounding each to three decimal places.

[1] $\coth 0.6 \approx$ 1.862 $\tanh^{-1} 0.6 \approx$ 0.693

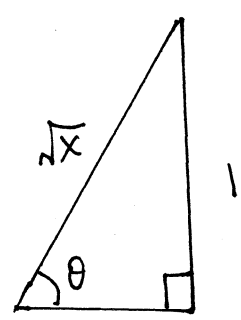
2. State the domain and range of $y = \arccos x$.

[1] domain = $[-1, 1]$ range = $[0, \pi]$

3. Simplify each of the following as much as possible by converting to algebraic form.

(a) $\cot(\underbrace{\operatorname{arccsc}\sqrt{x}}_{\theta})$

$\csc\theta = \sqrt{x}$



$\sqrt{(\sqrt{x})^2 - 1^2} = \sqrt{x-1}$

$\therefore \cot(\operatorname{arccsc}\sqrt{x}) = \cot\theta = \sqrt{x-1}$

[2]

(b) $\cosh(\ln x) - \sinh(\ln x)$

$$= \frac{e^{\ln x} + e^{-\ln x}}{2} - \frac{e^{\ln x} - e^{-\ln x}}{2} = e^{-\ln x} = e^{\ln x^{-1}} = \frac{1}{x}$$

[2]

4. Differentiate and simplify your answer: $y = \arctan(\sinh x)$

$$y' = \frac{\cosh x}{1 + \sinh^2 x} = \frac{\cosh x}{\cosh^2 x} = \frac{1}{\cosh x} = \operatorname{sech} x$$

[2]

5. Integrate: $\int \frac{x-5}{x^2+49}\,dx$

$$= \int \frac{x}{x^2+49}\,dx - 5\int \frac{1}{x^2+49}\,dx$$

[2]

$$= \frac{1}{2}\ln|x^2+49| - \frac{5}{7}\arctan\frac{x}{7} + C$$

6. Evaluate: $\int_0^{\pi/4} \frac{\sec^2 x}{\sqrt{4-\tan^2 x}}\,dx$ $= \int_0^1 \frac{du}{\sqrt{4-u^2}} = \arcsin\frac{u}{2}\Big|_0^1$

Let $u = \tan x$

$du = \sec^2 x\,dx$

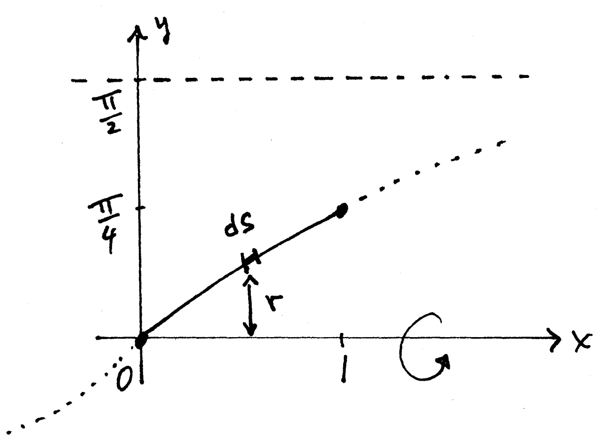
$x = 0 \Rightarrow u = 0$

$x = \frac{\pi}{4} \Rightarrow u = 1$

$= \arcsin\frac{1}{2} - \arcsin 0 = \frac{\pi}{6} - 0 = \frac{\pi}{6}$

[3]

7. Set up and simplify, **but do not evaluate**, a definite integral **in terms of y** representing the surface area of the surface of revolution obtained when the curve $y = \arctan x$ for $0 \le x \le 1$ is revolved about the ***x*-axis**.



[3]

$y = \arctan x \Rightarrow x = \tan y \Rightarrow \frac{dx}{dy} = \sec^2 y$

$ds = \sqrt{1+(\sec^2 y)^2}\,dy = \sqrt{1+\sec^4 y}\,dy$

$r = y$

$\Delta S = 2\pi r\,ds$ for $0 \le y \le \frac{\pi}{4}$

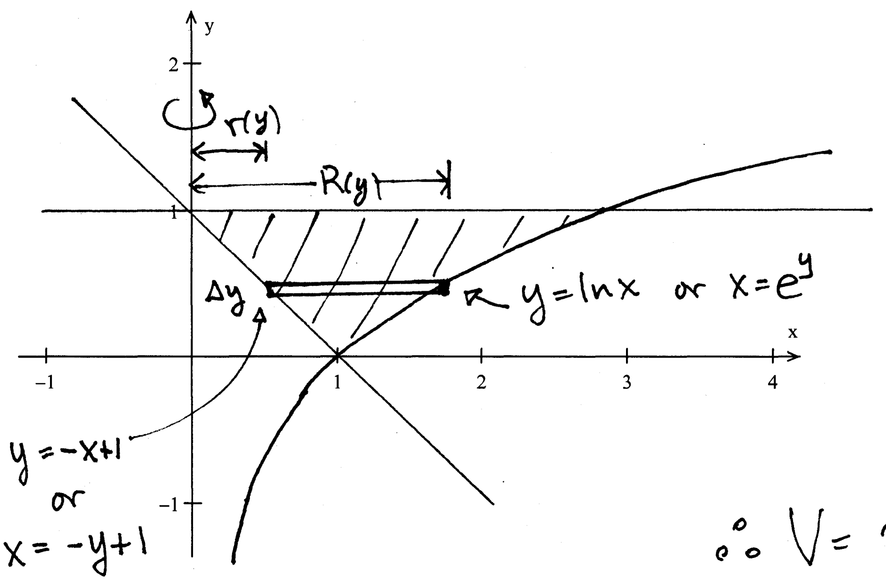
$$\therefore\ S = 2\pi \int_0^{\frac{\pi}{4}} y\sqrt{1+\sec^4 y}\,dy$$

8. Sketch and shade the region in the plane bounded by the curves

$$y = \ln x\,,\ y = -x + 1 \text{ and } y = 1,$$

and then set up, **but do not evaluate**, a definite integral (or sum of integrals) representing the volume of the solid formed by revolving the region about the ***y*-axis** using the **disk/washer** method.

[4]



for $0 \le y \le 1$

$\Delta V = \pi \left[R(y)^2 - r(y)^2 \right] \Delta y$

where $R(y) = e^y$

and $r(y) = -y+1$

$$\therefore\ V = \pi \int_0^1 \left(e^{2y} - (-y+1)^2 \right) dy$$

9. A large container is in the shape of a square pyramid having height 10 ft and a base measuring 10 ft on each side (see illustration). If the container is full of sand having weight density 100 lb/ft^3, then how much work is required to pump all of the sand out of the container through a hole at the top?

[5]

for $0 \le y \le 10$,

$\Delta F = 100 \Delta V$

$= 100 \left[L(y) \right]^2 \Delta y$

$= 100 (10-y)^2 \Delta y$

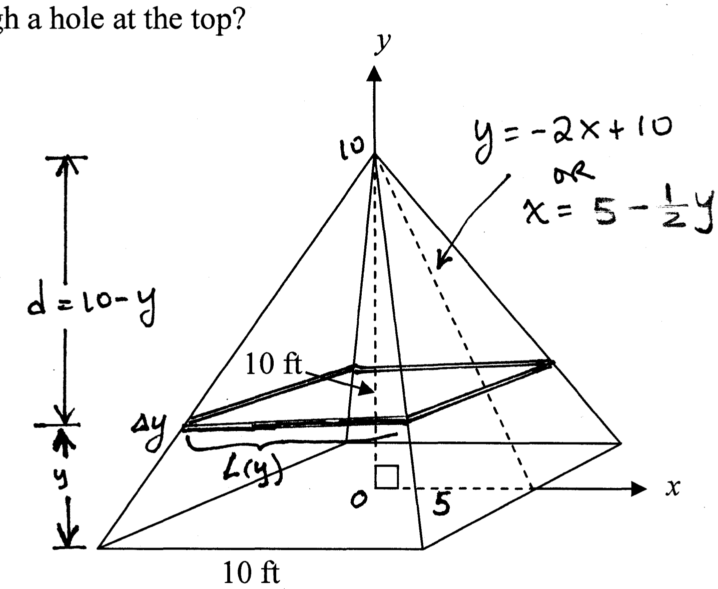
$\Delta W = \Delta F \cdot d = 100 (10-y)^3 \Delta y$

$W = 100 \int_0^{10} (10-y)^3\, dy$

$= -25 (10-y)^4 \Big|_0^{10}$

$= 0 + 250000$

$= 250\,000$ ft·lb



$L(y) = 2\left(5 - \tfrac{1}{2}y\right) = 10 - y$