Mathematics 101 Test #1A **Name:** SOLUTIONS

Instructor: George Ballinger **Section:** ___________

Term: Winter, 2018

Mark: $\frac{}{25}$

> The Sharp EL-531 calculator may be used on this test.
> Show all of your work in the space provided.
> The number of marks for each question is indicated in brackets.
> Give exact answers (no decimals) unless told otherwise.

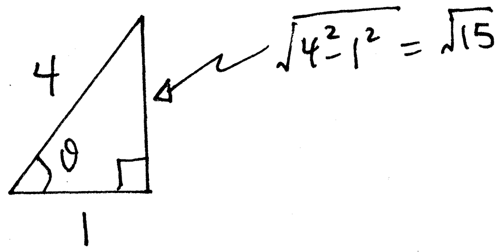
1. Solve: $\arctan\sqrt{x} = \operatorname{arcsec}4$

[3]

$$\sqrt{x} = \tan(\underbrace{\operatorname{arcsec}4}_{\theta})$$

Let $\theta = \operatorname{arcsec}4$

$\sec\theta = 4$



$$\sqrt{x} = \tan\theta = \frac{\sqrt{15}}{1} = \sqrt{15}$$

$\therefore x = 15$

2. Differentiate and simplify your answer: $y = x\arctan x - \ln\sqrt{1+x^2}$

[3]

$$y' = \frac{x}{1+x^2} + \arctan x - \frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{2\sqrt{1+x^2}} \cdot 2x$$

$$= \frac{x}{1+x^2} + \arctan x - \frac{x}{1+x^2} = \arctan x$$

3. Integrate: $\int \frac{1}{x^2+10x+61}\,dx$

[3]

$$= \int \frac{1}{x^2+10x+25+36}\,dx = \int \frac{1}{(x+5)^2+6^2}\,dx$$

$$= \frac{1}{6}\arctan\left(\frac{x+5}{6}\right) + C$$

4. Evaluate: $\displaystyle\int_0^{\ln\sqrt{3}} \frac{e^x}{\sqrt{4-e^{2x}}}\,dx \;=\; \int_1^{\sqrt{3}} \frac{du}{\sqrt{4-u^2}} \;=\; \arcsin\frac{u}{2}\Big|_1^{\sqrt{3}}$

Let $u = e^x$

$du = e^x dx$

$x = 0 \Rightarrow u = 1$

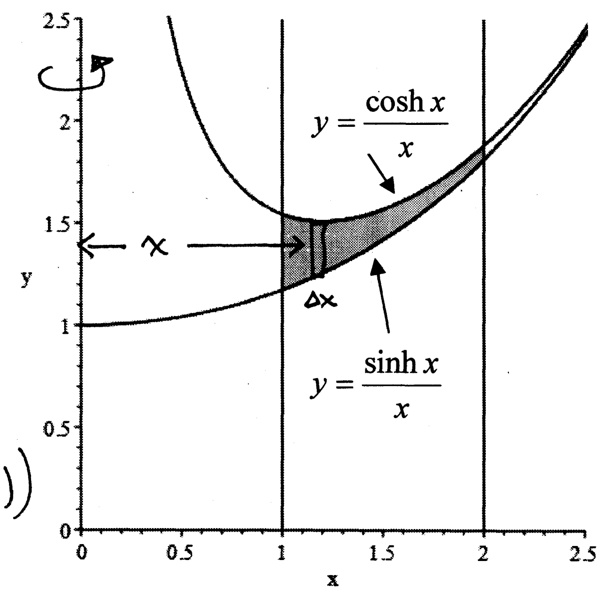
$x = \ln\sqrt{3} \Rightarrow u = \sqrt{3}$

$= \arcsin\frac{\sqrt{3}}{2} - \arcsin\frac{1}{2}$

$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

[4]

5. Use the shell method to find the volume of the solid of revolution obtained by revolving the region bounded by the curves $y = \dfrac{\cosh x}{x}$, $y = \dfrac{\sinh x}{x}$, $x = 1$ and $x = 2$, as shown, about the ***y*-axis**. Round your answer to four decimal places.



$\Delta V = 2\pi x \left(\frac{\cosh x}{x} - \frac{\sinh x}{x}\right)\Delta x$

for $1 \le x \le 2$

[3]

$V = 2\pi \int_1^2 (\cosh x - \sinh x)\,dx$

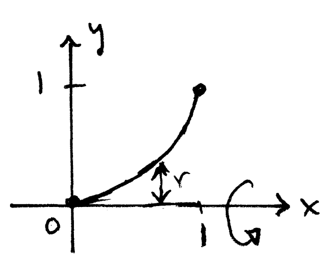
$= 2\pi \left[\sinh x - \cosh x\right]_1^2$

$= 2\pi \left((\sinh 2 - \cosh 2) - (\sinh 1 - \cosh 1)\right)$

≈ 1.4611

6. Find the surface area of the surface of revolution obtained when the curve $y = x^3$ for $0 \le x \le 1$ is revolved about the ***x*-axis**.

$y' = 3x^2$



$ds = \sqrt{1+(3x^2)^2}\,dx$

$= \sqrt{1+9x^4}\,dx$

$r(x) = x^3$

[4]

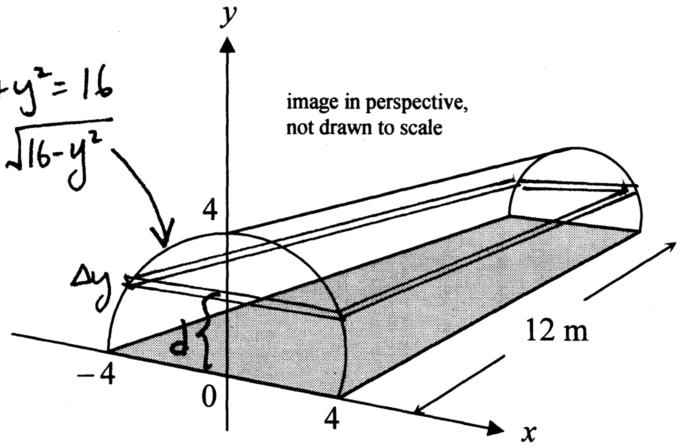
$S = 2\pi \int_0^1 x^3\sqrt{1+9x^4}\,dx = \frac{\pi}{18}\int_0^1 (1+9x^4)^{1/2}\, 36x^3\,dx$

$= \frac{\pi}{18}\cdot\frac{2}{3}(1+9x^4)^{3/2}\Big|_0^1 = \frac{\pi}{27}\left(10^{3/2} - 1\right)$

7. Each end of a 12 m long half-cylindrical tank is in the shape of a semicircle of radius 4 m as shown, where x and y are measured in meters. The bottom of the tank is sitting flat on the ground. The container is empty and is to be filled with West Texas Intermediate light crude oil weighing 8,100 N/m^3 by pumping it in through a hole in the bottom of the tank. Determine the amount of work done to completely fill the tank.

$x^2+y^2=16$

$x=\pm\sqrt{16-y^2}$



$$\Delta V = 2\sqrt{16-y^2}\cdot 12\cdot \Delta y$$
$$= 24\sqrt{16-y^2}\,\Delta y$$

$$\Delta F = 8100\,\Delta V = 8100\left(24\sqrt{16-y^2}\,\Delta y\right)$$
$$= 194400\sqrt{16-y^2}\,\Delta y$$

$$d = y$$

$$\Delta W = \Delta F\cdot d = 194400\,y\sqrt{16-y^2}\,\Delta y$$
for $0\le y\le 4$

$$W = 194400\int_0^4 y\sqrt{16-y^2}\,dy$$

[5]

$$= -97200\int_0^4 (16-y^2)^{1/2}(-2y)\,dy$$

$$= -97200\left(\tfrac{2}{3}\right)(16-y^2)^{3/2}\Big|_0^4$$

$$= 4,147,200\ \text{N}\cdot\text{m}$$