CAMOSUN

Name: SOLUTIONS

Mark: $\overline{}$ **25**

MATH 101 (Winter, 2023)

Test 1B

1. Let $f(x) = \arccos(\sin x)$ for $0 \leq x \leq \pi/2$.

   (a) (1 mark) Evaluate $f(0)$.

   $f(0) = \arccos(\sin 0) = \arccos 0 = \frac{\pi}{2}$

   (b) (2 marks) Find and simplify $f'(x)$.

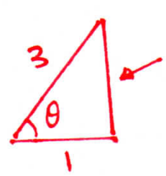
   $f'(x) = \frac{-1}{\sqrt{1-\sin^2 x}} \cdot \cos x = \frac{-\cos x}{\sqrt{\cos^2 x}} = \frac{-\cos x}{|\cos x|} = \frac{-\cos x}{\cos x} = -1$

   Since $\cos x \geq 0$ for $0 \leq x \leq \pi/2$

2. Let $\theta = \arccos(1/3)$.

   (a) (2 marks) Evaluate $\sin\theta$.

   $\cos\theta = \frac{1}{3}$



   $\sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$

   $\therefore \sin\theta = \frac{2\sqrt{2}}{3}$

   (b) (2 marks) Evaluate $\sin 2\theta$. *Hint: Use a double-angle formula.*

   $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{2\sqrt{2}}{3}\right)\left(\frac{1}{3}\right) = \frac{4\sqrt{2}}{9}$

3. (2 marks) Integrate $\displaystyle\int \frac{dx}{x^2 - 6x + 25}$.

$$\int \frac{dx}{x^2-6x+9+16} = \int \frac{dx}{(x-3)^2+16} = \frac{1}{4}\arctan\left(\frac{x-3}{4}\right) + C$$

4. (3 marks) Evaluate $\displaystyle\int_1^e \frac{dx}{x\sqrt{4 - (\ln x)^2}}$.

$$= \int_0^1 \frac{du}{\sqrt{4-u^2}} = \arcsin \frac{u}{2}\Big|_0^1$$

Let $u = \ln x$

$du = \frac{1}{x} dx$

$x = e \Rightarrow u = 1$

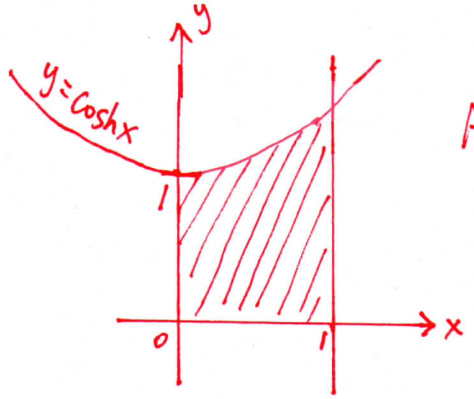
$x = 1 \Rightarrow u = 0$

$$= \arcsin \frac{1}{2} - \arcsin 0$$

$$= \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

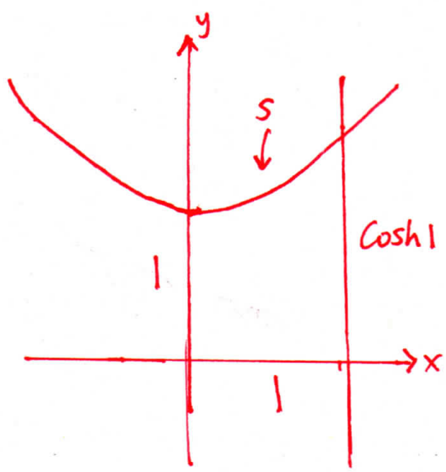
5. Consider the region in the plane bounded by the curves $y = \cosh x$, $y = 0$, $x = 0$, and $x = 1$.

(a) (3 marks) Find the area of the region. Include a sketch of the curves and shade the region.



$$A = \int_0^1 \cosh x\,dx = \sinh x\Big|_0^1 = \sinh 1 - \sinh 0 = \sinh 1$$

(b) (3 marks) Find the perimeter of the region.



$$S = \int_0^1 \sqrt{1+[f'(x)]^2}\,dx = \int_0^1 \sqrt{1+\sinh^2 x}\,dx$$

$$= \int_0^1 \sqrt{\cosh^2 x}\,dx = \int_0^1 \cosh x\,dx = \sinh 1$$

(as in part (a))

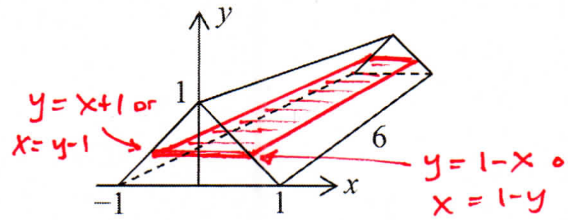
$$\therefore\ P = 1 + 1 + \cosh 1 + \sinh 1 = 2 + \cosh 1 + \sinh 1$$

(or $2 + e$)

(c) (2 marks) Set up, **but do not evaluate**, an integral representing the volume of the solid formed by revolving the region about the ***x*-axis**.

DISKS $\quad V = \pi \int_0^1 \cosh^2 x\,dx$

6. (5 marks) Each end of a 6 meter long tank is in the shape of an isosceles triangle measuring 2 meters across the bottom and having height 1 meter, as illustrated. If the tank is sitting flat on the ground and is full of water, which has weight density 9,800 N/m^3, then how much work is required to pump all the water out through a hole at the top of the tank?



For $0 \le y \le 1$,

$$\Delta V = [(1-y)-(y-1)]\cdot 6 \cdot \Delta y$$
$$= (2-2y)6\,\Delta y = 12(1-y)\Delta y$$

$$\Delta F = 9800\,\Delta V = 117600(1-y)\Delta y$$

$$d = 1-y$$

$$\Delta W = \Delta F \cdot d = 117600(1-y)\Delta y \cdot (1-y) = 117600(1-y)^2 \Delta y$$

$$\therefore W = 117600 \int_0^1 (1-y)^2\,dy = 117600\left[-\tfrac{1}{3}(1-y)^3\right]_0^1 = 117600\left(\tfrac{1}{3}\right) = 39200\text{ J}$$