Indeterminate and Determinate Forms of Limits

by George Ballinger

INDETERMINATE FORMS		
Form	**Example where limit exists**	**Example where limit does not exist**
$\frac{0}{0}$	$\lim_{x\to 0}\frac{\sin x}{x}=1$	$\lim_{x\to 0}\frac{\ln(x+1)}{x^2}$
$\frac{\infty}{\infty}$	$\lim_{x\to \infty}\frac{4x+1}{2x}=2$	$\lim_{x\to \infty}\frac{4x^2+1}{2x}$
$\infty-\infty$	$\lim_{x\to 0}\left(\csc^2 x-\frac{1}{x^2}\right)=\frac{1}{3}$	$\lim_{x\to 0}\left(\frac{1}{x^4}-\frac{1}{x^2}\right)$
$0\cdot\infty$	$\lim_{x\to \frac{\pi}{2}^-}\tan x\cos x=1$	$\lim_{x\to \frac{\pi}{2}^-}\tan x\sqrt{\frac{\pi}{2}-x}$
0^0	$\lim_{x\to 0^+}x^{\frac{1}{\ln x}}=e$	$\lim_{x\to 0^+}x^{\frac{-1}{\sqrt{\lvert\ln x\rvert}}}$
1^∞	$\lim_{x\to \infty}\left(1+\frac{1}{x}\right)^x=e$	$\lim_{x\to \infty}\left(1+\frac{1}{\sqrt{x}}\right)^x$
∞^0	$\lim_{x\to \infty}(10^x)^{\frac{1}{x}}=10$	$\lim_{x\to \infty}(10^{x^2})^{\frac{1}{x}}$

DETERMINATE FORMS (L represents a nonzero real number)			
Form	**Does limit exist?**		**Example**
$\infty+\infty$	no	limit is ∞	$\lim_{x\to 0}\left(\csc^2 x+\frac{1}{x^2}\right)=\infty$
0^∞	yes	limit is 0; assuming 0 is approached from the right	$\lim_{x\to \infty}\left(\frac{1}{x}\right)^x=0$
$0^{-\infty}$	no	limit is ∞; assuming 0 is approached from the right	$\lim_{x\to 0^+}x^{-\frac{1}{x^2}}=\infty$
$\frac{\infty}{0}$	no	limit may be ∞, $-\infty$, or neither; depending on how 0 is approached	$\lim_{x\to 0}\frac{\frac{1}{x^2}}{\sin x}$
$\frac{\infty}{L}$	no	limit is ∞ if $L>0$ and $-\infty$ if $L<0$	$\lim_{x\to \infty}\frac{e^x}{\arctan x}=\infty$
$\frac{L}{\infty}$	yes	limit is 0	$\lim_{x\to \infty}\frac{4}{\ln x}=0$
$\infty\cdot L$	no	limit is ∞ if $L>0$ and $-\infty$ if $L<0$	$\lim_{x\to \infty}x\ln\left(2+\frac{1}{x}\right)=\infty$
$L^\infty,\ 0<L<1$	yes	limit is 0	$\lim_{x\to \infty}\left(\frac{1}{2}\right)^x=0$
$L^\infty,\ L>1$	no	limit is ∞	$\lim_{x\to \infty}9^{\ln x}=\infty$