Review of Limits

(for more details see sec 1.2-1.5 and 3.5)

Let a, L and M represent real numbers.

Types of Limits:

$\lim_{x\to a} f(x)$	limit
$\lim_{x\to a^+} f(x)$	one-sided limit (from the right)
$\lim_{x\to a^-} f(x)$	one-sided limit (from the left)
$\lim_{x\to \infty} f(x)$	limit at infinity
$\lim_{x\to -\infty} f(x)$	limit at (negative) infinity

A limit either exists and equals L or it does not exist, in which case it may be ∞ or $-\infty$ (i.e. infinite limits) or neither.

Suppose $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then consider $\lim_{x\to a} \frac{f(x)}{g(x)}$.

1. If $M \neq 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ (see Theorem 1.2 (#4)).

2. If $M = 0$ and $L \neq 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist (d.n.e.); it may be $\pm\infty$ or neither.

3. If $M = L = 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ may or may not exist and is said to be in the *indeterminate form* $\frac{0}{0}$. Without additional information about $f(x)$ and $g(x)$, one cannot determine whether $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists, what it equals if it does exist, or whether it is an infinite limit if it does not exist. Consider these simple examples:

$$\lim_{x\to 0} \frac{x}{x} = \lim_{x\to 0} 1 = 1 \qquad \lim_{x\to 0} \frac{x^2}{x} = \lim_{x\to 0} x = 0$$

$$\lim_{x\to 0} \frac{x}{x^2} = \lim_{x\to 0} \frac{1}{x} \quad \text{d.n.e.} \qquad \lim_{x\to 0} \frac{x}{x^3} = \lim_{x\to 0} \frac{1}{x^2} = \infty \quad \text{(d.n.e.)}$$