Comparison Tests

Direct Comparison Test (DCT)

Suppose $0 < a_n \leq b_n$ for all $n \geq N$, where N is some positive integer.

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges.
2. If $\sum_{n=1}^{\infty} a_n$ diverges (to infinity), then $\sum_{n=1}^{\infty} b_n$ also diverges (to infinity).

Limit Comparison Test (LCT)

Suppose $a_n > 0$ and $b_n > 0$ for all $n \geq N$, where N is some positive integer. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L,$$

where L is finite and positive (i.e. $0 < L < \infty$), then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \sum_{n=1}^{\infty} b_n$$

either both converge or both diverge.