MATH 101 Test 3 Formulas

The essentials you should know

The following formulas are the most important formulas from sections 9.1-9.10. You should know all of these for Test 3 (and the final exam) in addition to the Test 1 & 2 formulas and MATH 100 and precalculus formulas, identities, etc. You should know all the tests for convergence/divergence of series except the Root test.

$$\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x), \text{ if } f(n) = a_n \text{ for all } n$$

$$\{a_n\} = \{a_1 + (n-1)d\} \qquad \text{[Arithmetic sequence]}$$

$$\{a_n\} = \{a_1 r^{n-1}\} \qquad \text{[Geometric sequence]}$$

$$\text{If } \sum_{n=1}^{\infty} a_n = A \text{ and } \sum_{n=1}^{\infty} b_n = B, \text{ then}$$

$$\sum_{n=1}^{\infty} ca_n = cA, \text{ for any real number } c, \text{ and } \sum_{n=1}^{\infty}(a_n \pm b_n) = A \pm B$$

$$\sum_{n=1}^{\infty}(b_n - b_{n+1}) = b_1 - \lim_{n\to\infty} b_{n+1} \qquad \text{[Telescoping series]}$$

$$\sum_{n=1}^{\infty}(b_n - b_{n+2}) = b_1 + b_2 - \lim_{n\to\infty}(b_{n+1} + b_{n+2}) \qquad \text{[Telescoping series variant]}$$

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \text{ if } 0 < |r| < 1 \qquad \text{[Geometric series]}$$

$$R_N = S - S_N, \text{ where } S \text{ is sum of series and } S_N \text{ is } N^{th} \text{ partial sum}$$

$$0 < R_N < \int_N^{\infty} f(x)\, dx \qquad \text{[Integral Test Remainder]}$$

$$|R_N| \le a_{N+1} \qquad \text{[Alternating Series Remainder]}$$

Taylor Polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

Taylor's Theorem with Remainder

$$f(x) = P_n(x) + R_n(x), \text{ where } R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1} \text{ for some } z \text{ between } x \text{ and } c$$

Taylor Series

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

$$\frac{d}{dx}\left[\sum_{n=0}^{\infty} a_n(x-c)^n\right] = \sum_{n=0}^{\infty}\left[\frac{d}{dx}\left(a_n(x-c)^n\right)\right] = \sum_{n=1}^{\infty} n a_n (x-c)^{n-1}$$

$$\int\left[\sum_{n=0}^{\infty} a_n(x-c)^n\right] dx = \sum_{n=0}^{\infty}\left[\int a_n(x-c)^n \, dx\right] = C + \sum_{n=0}^{\infty} \frac{a_n}{n+1}(x-c)^{n+1}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots = \sum_{n=0}^{\infty} x^n \qquad -1 < x < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}x^n \qquad -\infty < x < \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1} \qquad -\infty < x < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n} \qquad -\infty < x < \infty$$