CAMOSUN

Name: ______________________

Mark: $\overline{}$ **25**

MATH 101 (Fall, 2025)

Test 3

1. (2 marks) Answer whether the statements are True or False. No justification is required.

   (a) ____________ (b) ____________ (c) ____________ (d) ____________

   (a) If the sequence $\{a_n\}$ converges to zero, then $\sum_{n=1}^{\infty} a_n$ converges.

   (b) If $0 < a_n \leq b_n$ for all $n \geq 1$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges.

   (c) If $a_n \neq 0$ for all $n \geq 1$ and $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (d) If $a_n > 0$ and $b_n > 0$ for all $n \geq 1$, $\sum_{n=1}^{\infty} a_n$ converges, and $\lim_{n\to\infty} \frac{b_n}{a_n} = 2$, then $\sum_{n=1}^{\infty} b_n$ converges.

2. (3 marks) Determine whether the series converges conditionally, converges absolutely, or diverges. Identify which tests you are using and show that all of the conditions of the tests are satisfied.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$$

3. Find the sum of each convergent series.

   (a) (2 marks) $\sum_{n=1}^{\infty} \left(4^{1/n} - 4^{1/(n+1)}\right)$

   (b) (1 mark) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\pi^{2n}$

4. (3 marks) Determine whether the series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}$ converges or diverges. Identify which test you are using.

5. Let $f(x) = \cosh x$.

   (a) (2 marks) Find and simplify the 4^{th} Maclaurin polynomial, $P_4(x)$, for $f(x)$.

   (b) (1 mark) Use $P_4(x)$ from part (a) to approximate $\cosh(1/2)$. Express your answer as a fraction reduced to lowest terms.

   (c) (1 mark) Use the **definition** of Maclaurin series to find the Maclaurin series for $f(x)$. Express your answer using Σ-notation.

6. (3 marks) Use a power series for $\sin x$ to find a Taylor series for $f(x) = 5x \sin(5x^2)$ centered at $c = 0$. Express your answer using Σ-notation. What is the interval of convergence of the Taylor series?

7. Let $f(x) = \sum_{n=0}^{\infty} \frac{3}{(n+1)5^n}(x+1)^{n+1}$ be a power series function centered at $c = -1$.

(a) (1 mark) Find $f'(x)$, expressed in the form of a power series centered at $c = -1$.

(b) (1 mark) What type of series is the series in part (a)?

(c) (2 marks) Find the interval of convergence of the power series for $f'(x)$ found in part (a).

(d) (2 marks) Evaluate $f'(1)$.

(e) (1 mark) Find $\int f(x)\,dx$, expressed in the form of a power series centered at $c = -1$.