



Name: \_\_\_\_\_

Mark:         
**25**

**MATH 101 (Winter, 2023)**  
**Test 1B**

1. Let  $f(x) = \arccos(\sin x)$  for  $0 \leq x \leq \pi/2$ .

(a) (1 mark) Evaluate  $f(0)$ .

(b) (2 marks) Find and simplify  $f'(x)$ .

2. Let  $\theta = \arccos(1/3)$ .

(a) (2 marks) Evaluate  $\sin \theta$ .

(b) (2 marks) Evaluate  $\sin 2\theta$ . *Hint: Use a double-angle formula.*

3. (2 marks) Integrate  $\int \frac{dx}{x^2 - 6x + 25}$ .

4. (3 marks) Evaluate  $\int_1^e \frac{dx}{x\sqrt{4 - (\ln x)^2}}$ .

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5. Consider the region in the plane bounded by the curves  $y = \cosh x$ ,  $y = 0$ ,  $x = 0$ , and  $x = 1$ .
- (a) (3 marks) Find the area of the region. Include a sketch of the curves and shade the region.
- (b) (3 marks) Find the perimeter of the region.
- (c) (2 marks) Set up, **but do not evaluate**, an integral representing the volume of the solid formed by revolving the region about the  $x$ -axis.

6. (5 marks) Each end of a 6 meter long tank is in the shape of an isosceles triangle measuring 2 meters across the bottom and having height 1 meter, as illustrated. If the tank is sitting flat on the ground and is full of water, which has weight density  $9,800 \text{ N/m}^3$ , then how much work is required to pump all the water out through a hole at the top of the tank?

