



Name: _____

Mark:
25

MATH 101 (Winter, 2023)
Test 1A

1. Let $f(x) = \arcsin(\cos x)$ for $0 \leq x \leq \pi/2$.

(a) (1 mark) Evaluate $f(0)$.

(b) (2 marks) Find and simplify $f'(x)$.

2. Let $\theta = \arccos(2/3)$.

(a) (2 marks) Evaluate $\sin \theta$.

(b) (2 marks) Evaluate $\sin 2\theta$. *Hint: Use a double-angle formula.*

3. (2 marks) Integrate $\int \frac{dx}{x^2 + 8x + 25}$.

4. (3 marks) Evaluate $\int_1^e \frac{dx}{x\sqrt{2 - (\ln x)^2}}$.

-
5. Consider the region in the plane bounded by the curves $y = \cosh x$, $y = 0$, $x = 0$, and $x = 1$.
- (a) (3 marks) Find the area of the region. Include a sketch of the curves and shade the region.
- (b) (3 marks) Find the perimeter of the region.
- (c) (2 marks) Set up, **but do not evaluate**, an integral representing the volume of the solid formed by revolving the region about the **y -axis**.

6. (5 marks) Each end of a 6 meter long tank is in the shape of an isosceles triangle measuring 2 meters across the bottom and having height 1 meter, as illustrated. If the tank is sitting flat on the ground, then how much work is required to fill it with water, which has weight density $9,800 \text{ N/m}^3$, by pumping water into the tank through a hole at the bottom of the tank?

