Differentiation and Integration of Power Series

Define the function $f(x)$ by a power series centered at c according to

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n,$$

where the domain of f is the interval of convergence of the power series.

Then differentiating gives

$$f'(x) = \sum_{n=1}^{\infty} a_n n (x-c)^{n-1},$$

(note that indexing here starts at $n = 1$) and integrating gives

$$\int f(x)\, dx = C + \sum_{n=0}^{\infty} \frac{a_n}{n+1} (x-c)^{n+1}.$$

Both $f'(x)$ and $\int f(x)\, dx$ have the same radius R and interval of convergence as that of $f(x)$, except that in the case where $0 < R < \infty$, the behaviour at the endpoints $x = c \pm R$ may differ. Nevertheless,

1. if the series for $f(x)$ diverges at an endpoint, then so too does the series for $f'(x)$, and
2. if the series for $f(x)$ converges at an endpoint, then so too does the series for $\int f(x)\, dx$.