

Answers to Math 101 Final Exam Review Questions

1. $f'(x) = \frac{-x}{|x|\sqrt{1-x^2}}$

2. $-2\sqrt{7-6x-x^2} - 6\arcsin\left(\frac{x+3}{4}\right) + C$

3. a)

$$\begin{aligned} & \left(\frac{e^{2x} - e^{-2x}}{2}\right) \left(\frac{e^{5x} + e^{-5x}}{2}\right) \\ &= \frac{1}{2} \left(\frac{e^{7x} - e^{-7x}}{2} - \frac{e^{3x} - e^{-3x}}{2}\right) \\ &= \frac{1}{2} (\sinh 7x - \sinh 3x) \end{aligned}$$

b) $\frac{1}{14} \cosh 7x - \frac{1}{6} \cosh 3x + C$

4. a) $\frac{9}{2}$ b) $\frac{45p}{2}$ c) $\frac{72p}{5}$

5. a) $\sqrt{17} + \frac{1}{4} \ln(4 + \sqrt{17})$ b) $\frac{p}{6}(17^{\frac{1}{2}} - 1)$

6. tank half full when $h = \sqrt[3]{32}$, $W = 7.18 \text{ rg}$

7. $\left(\frac{p}{2}, \frac{p}{8}\right)$

8. a) $\sec x - \tan x + x + C$

b) $\frac{1}{3}x^3 \arcsin x + \frac{1}{3}\sqrt{1-x^2} - \frac{1}{9}(1-x^2)^{\frac{3}{2}} + C$

c) $\frac{3}{5} \ln|x-1| - \frac{3}{10} \ln(x^2+4) + \frac{7}{10} \arctan\left(\frac{x}{2}\right) + C$

d) $\ln|x + \sqrt{x^2-9}| - \frac{\sqrt{x^2-9}}{x} + C$

9. a) 1 b) 1

10.

$$\lim_{a \rightarrow 1^-} \int_0^a \frac{1}{(x-1)^3} dx + \lim_{b \rightarrow 1^+} \int_b^3 \frac{1}{(x-1)^3} dx +$$

$$\lim_{c \rightarrow \infty} \int_3^c \frac{1}{(x-1)^3} dx$$

The integral diverges.

11. a) Diverges using the LCT.

b) Diverges by the Ratio Test.

c) Diverges by the nth term test.

d) Integral Test shows convergence is not absolute. AST shows conditional convergence

12. Telescoping series. Sum = 1/5

13. $[-4, 6)$

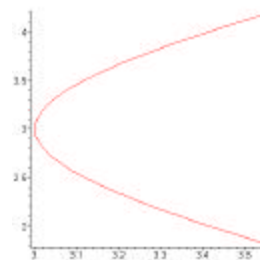
14.

$$P_4(x) = 1 + 2\left(x - \frac{p}{4}\right) + 2\left(x - \frac{p}{4}\right)^2 + \frac{8}{3}\left(x - \frac{p}{4}\right)^3 + \frac{10}{3}\left(x - \frac{p}{4}\right)^4$$

15. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{5n+1}}{5n+1} + C$

16. $\frac{1}{\sqrt{2p}} \sum_{n=0}^3 \frac{(-1)^n}{2^n n!(2n+1)} = .3412$, error < .00015

17. $(x-2)^2 - (y-3)^2 = 1$



right half of a hyperbola, orientation upwards

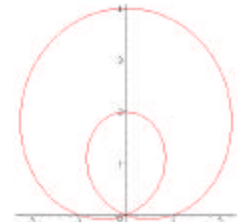
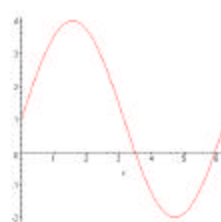
18. $\frac{dy}{dx} = \frac{1+2\sin t}{\cos t}$

Hor. Tan at $\left(\frac{1}{2}, \frac{7p}{6} + \sqrt{3}\right)$, $\left(\frac{1}{2}, \frac{11p}{6} - \sqrt{3}\right)$

Ver. Tan at $\left(2, \frac{p}{2}\right)$, $\left(0, \frac{3p}{2}\right)$

19. $s = 2(5^{\frac{3}{2}} - 1)$

20.



21. a) $(x^2 - y^2)^{\frac{3}{2}} = x^2 - y^2$

b) $r = \frac{b}{\sin q - m \cos q}$

22. $\frac{5p}{24} - \frac{\sqrt{3}}{4}$

23. a) 8 b) $\frac{32p}{5}$