B Integration Tables

Forms Involving u^n

1. $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1} + C,\ n \neq -1$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C$

Forms Involving $a + bu$

3. $\displaystyle\int \frac{u}{a+bu}\,du = \frac{1}{b^2}\big(bu - a\ln|a+bu|\big) + C$

4. $\displaystyle\int \frac{u}{(a+bu)^2}\,du = \frac{1}{b^2}\left(\frac{a}{a+bu} + \ln|a+bu|\right) + C$

5. $\displaystyle\int \frac{u}{(a+bu)^n}\,du = \frac{1}{b^2}\left[\frac{-1}{(n-2)(a+bu)^{n-2}} + \frac{a}{(n-1)(a+bu)^{n-1}}\right] + C,\quad n \neq 1, 2$

6. $\displaystyle\int \frac{u^2}{a+bu}\,du = \frac{1}{b^3}\left[-\frac{bu}{2}(2a - bu) + a^2\ln|a+bu|\right] + C$

7. $\displaystyle\int \frac{u^2}{(a+bu)^2}\,du = \frac{1}{b^3}\left(bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$

8. $\displaystyle\int \frac{u^2}{(a+bu)^3}\,du = \frac{1}{b^3}\left[\frac{2a}{a+bu} - \frac{a^2}{2(a+bu)^2} + \ln|a+bu|\right] + C$

9. $\displaystyle\int \frac{u^2}{(a+bu)^n}\,du = \frac{1}{b^3}\left[\frac{-1}{(n-3)(a+bu)^{n-3}} + \frac{2a}{(n-2)(a+bu)^{n-2}} - \frac{a^2}{(n-1)(a+bu)^{n-1}}\right] + C,\quad n \neq 1, 2, 3$

10. $\displaystyle\int \frac{1}{u(a+bu)}\,du = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

11. $\displaystyle\int \frac{1}{u(a+bu)^2}\,du = \frac{1}{a}\left(\frac{1}{a+bu} + \frac{1}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

12. $\displaystyle\int \frac{1}{u^2(a+bu)}\,du = -\frac{1}{a}\left(\frac{1}{u} + \frac{b}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

13. $\displaystyle\int \frac{1}{u^2(a+bu)^2}\,du = -\frac{1}{a^2}\left[\frac{a+2bu}{u(a+bu)} + \frac{2b}{a}\ln\left|\frac{u}{a+bu}\right|\right] + C$

Forms Involving $a + bu + cu^2,\ b^2 \neq 4ac$

14. $$\int \frac{1}{a + bu + cu^2}\,du = \begin{cases} \dfrac{2}{\sqrt{4ac - b^2}} \arctan \dfrac{2cu + b}{\sqrt{4ac - b^2}} + C, & b^2 < 4ac \\ \dfrac{1}{\sqrt{b^2 - 4ac}} \ln \left| \dfrac{2cu + b - \sqrt{b^2 - 4ac}}{2cu + b + \sqrt{b^2 - 4ac}} \right| + C, & b^2 > 4ac \end{cases}$$

15. $$\int \frac{u}{a + bu + cu^2}\,du = \frac{1}{2c}\left(\ln|a + bu + cu^2| - b \int \frac{1}{a + bu + cu^2}\,du\right)$$

Forms Involving $\sqrt{a + bu}$

16. $$\int u^n \sqrt{a + bu}\,du = \frac{2}{b(2n + 3)}\left[u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu}\,du\right]$$

17. $$\int \frac{1}{u\sqrt{a + bu}}\,du = \begin{cases} \dfrac{1}{\sqrt{a}} \ln \left| \dfrac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, & a > 0 \\ \dfrac{2}{\sqrt{-a}} \arctan \sqrt{\dfrac{a + bu}{-a}} + C, & a < 0 \end{cases}$$

18. $$\int \frac{1}{u^n \sqrt{a + bu}}\,du = \frac{-1}{a(n - 1)}\left[\frac{\sqrt{a + bu}}{u^{n-1}} + \frac{(2n - 3)b}{2} \int \frac{1}{u^{n-1}\sqrt{a + bu}}\,du\right],\ n \neq 1$$

19. $$\int \frac{\sqrt{a + bu}}{u}\,du = 2\sqrt{a + bu} + a \int \frac{1}{u\sqrt{a + bu}}\,du$$

20. $$\int \frac{\sqrt{a + bu}}{u^n}\,du = \frac{-1}{a(n - 1)}\left[\frac{(a + bu)^{3/2}}{u^{n-1}} + \frac{(2n - 5)b}{2} \int \frac{\sqrt{a + bu}}{u^{n-1}}\,du\right],\ n \neq 1$$

21. $$\int \frac{u}{\sqrt{a + bu}}\,du = \frac{-2(2a - bu)}{3b^2}\sqrt{a + bu} + C$$

22. $$\int \frac{u^n}{\sqrt{a + bu}}\,du = \frac{2}{(2n + 1)b}\left(u^n \sqrt{a + bu} - na \int \frac{u^{n-1}}{\sqrt{a + bu}}\,du\right)$$

Forms Involving $a^2 \pm u^2,\ a > 0$

23. $$\int \frac{1}{a^2 + u^2}\,du = \frac{1}{a} \arctan \frac{u}{a} + C$$

24. $$\int \frac{1}{u^2 - a^2}\,du = -\int \frac{1}{a^2 - u^2}\,du = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$$

25. $$\int \frac{1}{(a^2 \pm u^2)^n}\,du = \frac{1}{2a^2(n - 1)}\left[\frac{u}{(a^2 \pm u^2)^{n-1}} + (2n - 3) \int \frac{1}{(a^2 \pm u^2)^{n-1}}\,du\right],\ n \neq 1$$

Forms Involving $\sqrt{u^2 \pm a^2},\ a > 0$

26. $$\int \sqrt{u^2 \pm a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \pm a^2 \ln|u + \sqrt{u^2 \pm a^2}|\right) + C$$

27. $$\int u^2 \sqrt{u^2 \pm a^2}\,du = \frac{1}{8}\left[u(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - a^4 \ln|u + \sqrt{u^2 \pm a^2}|\right] + C$$

28. $$\int \frac{\sqrt{u^2 + a^2}}{u}\,du = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

29. $\displaystyle\int \frac{\sqrt{u^2 - a^2}}{u}\,du = \sqrt{u^2 - a^2} - a \operatorname{arcsec}\frac{|u|}{a} + C$

30. $\displaystyle\int \frac{\sqrt{u^2 \pm a^2}}{u^2}\,du = \frac{-\sqrt{u^2 \pm a^2}}{u} + \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

31. $\displaystyle\int \frac{1}{\sqrt{u^2 \pm a^2}}\,du = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

32. $\displaystyle\int \frac{1}{u\sqrt{u^2 + a^2}}\,du = \frac{-1}{a}\ln\left|\frac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

33. $\displaystyle\int \frac{1}{u\sqrt{u^2 - a^2}}\,du = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

34. $\displaystyle\int \frac{u^2}{\sqrt{u^2 \pm a^2}}\,du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \mp a^2 \ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

35. $\displaystyle\int \frac{1}{u^2\sqrt{u^2 \pm a^2}}\,du = \mp\frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$

36. $\displaystyle\int \frac{1}{(u^2 \pm a^2)^{3/2}}\,du = \frac{\pm u}{a^2\sqrt{u^2 \pm a^2}} + C$

Forms Involving $\sqrt{a^2 - u^2},\ a > 0$

37. $\displaystyle\int \sqrt{a^2 - u^2}\,du = \frac{1}{2}\left(u\sqrt{a^2 - u^2} + a^2 \arcsin\frac{u}{a}\right) + C$

38. $\displaystyle\int u^2\sqrt{a^2 - u^2}\,du = \frac{1}{8}\left[u(2u^2 - a^2)\sqrt{a^2 - u^2} + a^4 \arcsin\frac{u}{a}\right] + C$

39. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u}\,du = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

40. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u^2}\,du = \frac{-\sqrt{a^2 - u^2}}{u} - \arcsin\frac{u}{a} + C$

41. $\displaystyle\int \frac{1}{\sqrt{a^2 - u^2}}\,du = \arcsin\frac{u}{a} + C$

42. $\displaystyle\int \frac{1}{u\sqrt{a^2 - u^2}}\,du = \frac{-1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

43. $\displaystyle\int \frac{u^2}{\sqrt{a^2 - u^2}}\,du = \frac{1}{2}\left(-u\sqrt{a^2 - u^2} + a^2 \arcsin\frac{u}{a}\right) + C$

44. $\displaystyle\int \frac{1}{u^2\sqrt{a^2 - u^2}}\,du = \frac{-\sqrt{a^2 - u^2}}{a^2 u} + C$

45. $\displaystyle\int \frac{1}{(a^2 - u^2)^{3/2}}\,du = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$

Forms Involving $\sin u$ or $\cos u$

46. $\int \sin u\,du = -\cos u + C$

47. $\int \cos u\,du = \sin u + C$

48. $\int \sin^2 u\,du = \frac{1}{2}(u - \sin u \cos u) + C$

49. $\int \cos^2 u\,du = \frac{1}{2}(u + \sin u \cos u) + C$

50. $\int \sin^n u\,du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n}\int \sin^{n-2} u\,du$

51. $\int \cos^n u\,du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n}\int \cos^{n-2} u\,du$

52. $\int u \sin u\,du = \sin u - u \cos u + C$

53. $\int u \cos u\,du = \cos u + u \sin u + C$

54. $\int u^n \sin u\,du = -u^n \cos u + n\int u^{n-1} \cos u\,du$

55. $\int u^n \cos u\,du = u^n \sin u - n\int u^{n-1} \sin u\,du$

56. $\int \frac{1}{1 \pm \sin u}\,du = \tan u \mp \sec u + C$

57. $\int \frac{1}{1 \pm \cos u}\,du = -\cot u \pm \csc u + C$

58. $\int \frac{1}{\sin u \cos u}\,du = \ln|\tan u| + C$

Forms Involving $\tan u$, $\cot u$, $\sec u$, $\csc u$

59. $\int \tan u\,du = -\ln|\cos u| + C$

60. $\int \cot u\,du = \ln|\sin u| + C$

61. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

62. $\int \csc u\,du = \ln|\csc u - \cot u| + C$ or $\int \csc u\,du = -\ln|\csc u + \cot u| + C$

63. $\int \tan^2 u\,du = -u + \tan u + C$

64. $\int \cot^2 u\,du = -u - \cot u + C$

65. $\int \sec^2 u\,du = \tan u + C$

66. $\int \csc^2 u\,du = -\cot u + C$

67. $\int \tan^n u\,du = \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u\,du,\ n \neq 1$

68. $\int \cot^n u\,du = -\frac{\cot^{n-1} u}{n-1} - \int (\cot^{n-2} u)\,du,\ n \neq 1$

69. $\int \sec^n u\,du = \frac{\sec^{n-2} u \tan u}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} u\,du,\ n \neq 1$

70. $\int \csc^n u\,du = -\frac{\csc^{n-2} u \cot u}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} u\,du,\ n \neq 1$

71. $\int \frac{1}{1 \pm \tan u}\,du = \frac{1}{2}\big(u \pm \ln|\cos u \pm \sin u|\big) + C$

72. $\int \frac{1}{1 \pm \cot u}\,du = \frac{1}{2}\big(u \mp \ln|\sin u \pm \cos u|\big) + C$

73. $\int \frac{1}{1 \pm \sec u}\,du = u + \cot u \mp \csc u + C$

74. $\int \frac{1}{1 \pm \csc u}\,du = u - \tan u \pm \sec u + C$

Forms Involving Inverse Trigonometric Functions

75. $\int \arcsin u\,du = u\arcsin u + \sqrt{1-u^2} + C$

76. $\int \arccos u\,du = u\arccos u - \sqrt{1-u^2} + C$

77. $\int \arctan u\,du = u\arctan u - \ln\sqrt{1+u^2} + C$

78. $\int \operatorname{arccot} u\,du = u\operatorname{arccot} u + \ln\sqrt{1+u^2} + C$

79. $\int \operatorname{arcsec} u\,du = u\operatorname{arcsec} u - \ln\left|u + \sqrt{u^2-1}\right| + C$

80. $\int \operatorname{arccsc} u\,du = u\operatorname{arccsc} u + \ln\left|u + \sqrt{u^2-1}\right| + C$

Forms Involving e^u

81. $\int e^u\,du = e^u + C$

82. $\int ue^u\,du = (u-1)e^u + C$

83. $\int u^n e^u\,du = u^n e^u - n\int u^{n-1}e^u\,du$

84. $\int \frac{1}{1+e^u}\,du = u - \ln(1+e^u) + C$

85. $\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$

86. $\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$

Forms Involving $\ln u$

87. $\int \ln u\,du = u(-1+\ln u) + C$

88. $\int u\ln u\,du = \frac{u^2}{4}(-1+2\ln u) + C$

89. $\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[-1+(n+1)\ln u] + C,\ n \neq -1$

90. $\int (\ln u)^2\,du = u\left[2 - 2\ln u + (\ln u)^2\right] + C$

91. $\int (\ln u)^n\,du = u(\ln u)^n - n\int (\ln u)^{n-1}\,du$

Forms Involving Hyperbolic Functions

92. $\int \cosh u\,du = \sinh u + C$

93. $\int \sinh u\,du = \cosh u + C$

94. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

95. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

96. $\int \operatorname{sech} u\tanh u\,du = -\operatorname{sech} u + C$

97. $\int \operatorname{csch} u\coth u\,du = -\operatorname{csch} u + C$

Forms Involving Inverse Hyperbolic Functions (in logarithmic form)

98. $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

99. $\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$

100. $\int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a}\ln\frac{a + \sqrt{a^2 \pm u^2}}{|u|} + C$