Hyperbolic Formulas

Definitions of Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} \qquad \operatorname{sech} x = \frac{1}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}$$

Hyperbolic Identities

$$\cosh^2 x - \sinh^2 x = 1 \qquad \tanh^2 x + \operatorname{sech}^2 x = 1 \qquad \coth^2 x - \operatorname{csch}^2 x = 1$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y \qquad \cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\sinh^2 x = \frac{-1 + \cosh 2x}{2} \qquad \cosh^2 x = \frac{1 + \cosh 2x}{2}$$

$$\sinh 2x = 2 \sinh x \cosh x \qquad \cosh 2x = \cosh^2 x + \sinh^2 x$$

Derivatives and Integrals of Hyperbolic Functions

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x \qquad \frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$$

$$\frac{d}{dx}[\coth x] = -\operatorname{csch}^2 x \qquad \frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x \qquad \frac{d}{dx}[\operatorname{csch} x] = -\operatorname{csch} x \coth x$$

$$\int \cosh x \; dx = \sinh x + C \qquad \int \sinh x \; dx = \cosh x + C \qquad \int \operatorname{sech}^2 x \; dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x \; dx = -\coth x + C \qquad \int \operatorname{sech} x \tanh x \; dx = -\operatorname{sech} x + C \qquad \int \operatorname{csch} x \coth x \; dx = -\operatorname{csch} x + C$$

Logarithmic Form of Inverse Hyperbolic Functions

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}) \qquad \cosh^{-1} x = \ln(x + \sqrt{x^2-1})$$

$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x} \qquad \coth^{-1} x = \frac{1}{2}\ln\frac{x+1}{x-1}$$

$$\operatorname{sech}^{-1} x = \ln\frac{1+\sqrt{1-x^2}}{x} \qquad \operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right)$$

Derivatives of Inverse Hyperbolic Functions

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{x^2+1}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2}$$

$$\frac{d}{dx}[\coth^{-1} x] = \frac{1}{1-x^2} \qquad \frac{d}{dx}[\operatorname{sech}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}} \qquad \frac{d}{dx}[\operatorname{csch}^{-1} x] = \frac{-1}{|x|\sqrt{1+x^2}}$$

Integrals Involving Inverse Hyperbolic Functions

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\frac{x}{a} + C = \ln(x + \sqrt{x^2+a^2}) + C$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\frac{x}{a} + C = \ln(x + \sqrt{x^2-a^2}) + C$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{a}\tanh^{-1}\frac{x}{a} + C = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$