CAMOSUN

Name: ______________________

Mark: $\frac{\quad}{25}$

MATH 101
Assignment 6

1. (2 marks) Test the series

$$\sum_{n=1}^{\infty} \frac{n^n n!}{(2n)!}$$

for convergence or divergence. Be sure to name the test you are using and verify that all the conditions of the test are satisfied. You may use the fact that

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$$

without having to prove it.

2. (2 marks) Find the 3^{rd} Taylor polynomial for $f(x) = \tan x$ centered at $c = \pi/4$.

3. (2 marks) Let $f(x) = 1 - \cos x$. Find and simplify the 4^{th} Maclaurin polynomial for $f(x)$ and use it to approximate $f(0.5)$. Then use Taylor's Theorem to estimate the size of the error.

4. Define $f(x) = \sum_{n=1}^{\infty} \frac{(x-3)^{2n}}{n^2 4^n}$.

   (a) (3 marks) Find the radius and interval of convergence of $f(x)$.

(b) (3 marks) Find power series representations, using Σ-notation, for $f'(x)$ and $\int f(x)\,dx$ and find each of their intervals of convergence.

5. (2 marks) Find a power series for $f(x) = \dfrac{3}{2x+5}$ centered at $c = 4$ and determine its interval of convergence.

6. (a) (1 mark) Find a power series for $f(x) = \dfrac{1}{(1-x)^2}$ centered at $c = 0$ and determine its interval of convergence.

(b) (2 marks) Using your answer from part (a), find a power series for $g(x) = \dfrac{4x^3}{(1-4x^2)^2}$ centered at $c = 0$ and determine its interval of convergence.

(c) (1 mark) Using your answer from part (b), find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4^{n+1}}$.

7. (2 marks) Use the definition of a Maclaurin series to find the Maclaurin series for $\sinh x$.

8. (2 marks) Using known power series and a trigonometric identity, find a Taylor series centered at $c = 0$ for the function $f(x) = x(1 - 2\sin^2 x)$.

9. (3 marks) Consider the definite integral

$$\int_0^{1/2} x^2 e^{-x^2}\,dx.$$

(a) Express the integral in the form of a power series using Σ-notation.

(b) Use the power series from part (a) to approximate the value of the integral with an error of less than 0.001. Justify why your answer is sufficiently accurate.