CAMOSUN

Name: ______________________

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MATH 101
Assignment 5

1. (1 mark) Write an expression for the n^{th} term of the sequence whose first few terms are

$$1, -\frac{3}{2}, \frac{5}{4}, -\frac{7}{8}, \frac{9}{16}, -\frac{11}{32}, \ldots \qquad a_n = \underline{\qquad\qquad}$$

2. Determine the convergence or divergence of the sequence $\{a_n\}$ whose n^{th} term is given.

(a) (1 mark) $a_n = \dfrac{2n}{5n + 3\sqrt{n}}$

(b) (1 mark) $a_n = \cos 2n\pi$

3. (1 mark) Find the sum of the convergent series $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{3}\left(\frac{2}{3}\right)^n + \frac{2}{3}\left(\frac{1}{3}\right)^n \right)$.

4. Let $a_n = \dfrac{4(n+1)(2n)!}{(2n+3)!}$ for $n \geq 1$.

   (a) (1 mark) Simplify a_n by eliminating factorials and reducing to lowest terms.

   (b) (1 mark) Determine whether the sequence $\{a_n\}$ converges or diverges. If it converges, find its limit.

   (c) (1 mark) What, if anything, can the n^{th} Term Test for Divergence allow you to conclude about convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty} a_n$?

   (d) (2 marks) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges or diverges. If it converges, find its sum. *[Hint: Convert the series into a telescoping series.]*

5. Test each series for convergence or divergence using only tests from sec 9.2-9.3 (n^{th} Term Test, Geometric Series Test, Telescoping Series Test, p-Series Test, or Integral Test). Be sure to name the test you are using and verify that all the conditions of the test are satisfied.

   (a) (1 mark) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

   (b) (1 mark) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{10^n\sqrt{10^n}}$

   (c) (1 mark) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$

6. (4 marks) Consider the series $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^5}$.

   (a) Verify that the series satisfies all of the conditions of the Integral Test.

   (b) Use the Integral Test to determine whether the series converges or diverges.

7. (2 marks) The following series converges according to the Integral Test. Approximate its sum using four terms. Then calculate an estimate of the maximum error for your approximation and give an interval in which the exact sum S must lie. Round all your answers to six decimal places.

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)^6}$$

8. Test each series for convergence or divergence. Be sure to name the test you are using and verify that all the conditions of the test are satisfied.

   (a) (1 mark) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^4 n}{n^4}$

   (b) (2 marks) $\displaystyle\sum_{n=1}^{\infty} \sin \frac{1}{n}$

9. Consider the series
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{3n+1}}.$$

   (a) (3 marks) Determine whether the series converges absolutely, converges conditionally, or diverges.

   (b) (1 mark) If the series converges, determine the number of terms required to approximate its sum with an error of no more than 0.01. If the series diverges, then what can you say about the limit $\lim_{n\to\infty} S_n$, where S_n denotes the n^{th} partial sum of the series?