

The Sharp EL-531W calculator may be used on this test.

Show all of your work in the space provided.

The number of marks for each question is indicated in brackets.



1. Evaluate the following integrals.

(a)  $\int \frac{x^3}{(x^4 - 4)^6} dx$

[3]

(b)  $\int x \sqrt[3]{x-2} dx$

[3]

2. Find the **exact** value of the definite integral  $\int_{\pi/3}^{\pi/2} \sin^3 \frac{x}{2} \cos \frac{x}{2} dx$ .

[3]

3. Let  $f(x) = e^{-x/2}$ .

- Find the average value of  $f(x)$  on the interval  $[0, 3]$ . Round your answer to four decimal places.
- Find the value(s) of  $x$  in the interval  $[0, 3]$  for which the function equals its average value. Round your answer(s) to four decimal places.
- Sketch the graph of  $f(x)$  on the interval  $[0, 3]$  and label the average value and corresponding  $x$ -value(s) on your graph.

[5]

4. If the Trapezoidal Rule were used to approximate the integral  $\int_2^4 \frac{1}{\sqrt{x}} dx$  with  $n = 24$ , then calculate the **exact** value of  $f(x_3)$  in the formula for the Trapezoidal Rule. Do not compute the entire approximation.

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

[2]

5. Evaluate  $\int_{-5}^5 (x+5)^3 dx$  by using the limit of a Riemann sum definition of a definite integral.

[3]

6. A particle moves along the  $x$ -axis at a velocity  $v(t) = 6\sec 2t \tan 2t$  for  $t \geq 0$ . At time  $t = 0$ , its position is  $x = 4$ . Find the position of the particle at time  $t = 1/2$ . Round your answer to three decimal places.

[3]

7. Use logarithmic differentiation to differentiate  $y = x^{\cos x}$ .

[3]