CAMOSUN COLLEGE

Mathematics 100 Test #3A **Name:** SOLUTIONS

Instructor: George Ballinger **Section:** ____________

Term: Fall, 2011

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.

Mark: ____ / 25

1. Find the following indefinite integrals.

(a) $\int\left(8x^2-7x+2-e^x+\frac{3}{x}\right)dx$

$= \frac{8}{3}x^3-\frac{7}{2}x^2+2x-e^x+3\ln|x|+C$

[2]

(b) $\int 12\sin 5\theta\cos^3 5\theta\, d\theta$

$= 12\int \cos^3 5\theta \sin 5\theta\, d\theta$

$= -\frac{12}{5}\int(\cos 5\theta)^3(-5\sin 5\theta)\,d\theta$

[3] $= -\frac{12}{5}\cdot\frac{1}{4}(\cos 5\theta)^4+C$

$= -\frac{3}{5}\cos^4 5\theta + C$

OR

Let $u=\cos 5\theta$

$du = -5\sin 5\theta\, d\theta$

$\sin 5\theta\, d\theta = -\frac{1}{5}du$

$\int 12\sin 5\theta\cos^3 5\theta\, d\theta$

$= \int 12u^3\left(-\frac{1}{5}\right)du = -\frac{12}{5}\int u^3\,du$

$= -\frac{12}{5}\cdot\frac{1}{4}u^4 + C = -\frac{3}{5}\cos^4 5\theta + C$

(c) $\int 7x\sqrt{6-x}\,dx$

Let $u=6-x \rightarrow x=6-u$

$du=-dx \rightarrow dx=-du$

$\int 7x\sqrt{6-x}\,dx = \int 7(6-u)u^{1/2}(-1)\,du$

[3] $= \int 7u^{1/2}(u-6)\,du = \int(7u^{3/2}-42u^{1/2})\,du$

$= 7\cdot\frac{2}{5}u^{5/2}-42\cdot\frac{2}{3}u^{3/2}+C$

$= \frac{14}{5}(6-x)^{5/2}-28(6-x)^{3/2}+C$

OR

Let $u=\sqrt{6-x}$

$u^2=6-x \Rightarrow x=6-u^2$

$2u\,du=-dx \rightarrow dx=-2u\,du$

$\int 7x\sqrt{6-x}\,dx = \int 7(6-u^2)u(-2u)\,du$

$= \int 14u^2(u^2-6)\,du = \int(14u^4-84u^2)\,du$

$= \frac{14}{5}u^5-28u^3+C$

$= \frac{14}{5}(6-x)^{5/2}-28(6-x)^{3/2}+C$

2. Let $f(x) = 4x^3 + 3x + 5$ and note that $f(1) = 12$. Find $(f^{-1})(12)$ and $(f^{-1})'(12)$.

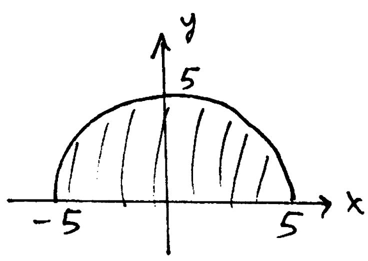
$f'(x) = 12x^2 + 3 > 0$ for all x ; $\therefore$ f is increasing and has an inverse.

$(f^{-1})(12) = 1$

$(f^{-1})'(12) = \dfrac{1}{f'(1)} = \dfrac{1}{15}$

[2]

3. Sketch the region of the *xy*-plane whose area is represented by the integral $\int_{-5}^{5} \sqrt{25 - x^2}\, dx$ and then evaluate the integral by using a geometry formula.



$\int_{-5}^{5} \sqrt{25-x^2}\, dx$ = area of semicircle

$= \frac{1}{2} \cdot (\pi \cdot 5^2) = \frac{25\pi}{2}$

[2]

4. Let $f(x) = \int_1^{x^2} \sin\sqrt{t}\, dt$ and find $f'(\pi/2)$.

$f'(x) = \sin\sqrt{x^2} \cdot 2x = 2x \sin x$

[2] $f'(\pi/2) = 2 \cdot \frac{\pi}{2} \cdot \sin\frac{\pi}{2} = \pi$

5. Evaluate $\int_{-4}^{2}(x+4)^2\,dx$ by using the limit of a Riemann sum definition of a definite integral:

$a=-4,\ b=2,\ \Delta x=\frac{2-(-4)}{n}=\frac{6}{n},\quad f(x)=(x+4)^2$

$$\int_{-4}^{2}(x+4)^2dx=\lim_{n\to\infty}\sum_{i=1}^{n}f\left(-4+i\left(\tfrac{6}{n}\right)\right)\cdot\frac{6}{n}=\lim_{n\to\infty}\sum_{i=1}^{n}\left(-4+\frac{6i}{n}+4\right)^2\cdot\frac{6}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{6i}{n}\right)^2\cdot\frac{6}{n}=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{216i^2}{n^3}=\lim_{n\to\infty}\frac{216}{n^3}\sum_{i=1}^{n}i^2$$

$$=\lim_{n\to\infty}\frac{216}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}=\lim_{n\to\infty}\frac{36}{n^2}(2n^2+3n+1)$$

[3] $$=\lim_{n\to\infty}\left(72+\frac{108}{n}+\frac{36}{n^2}\right)=72$$

6. Use logarithmic differentiation to differentiate $y=(\sin x)^{e^x}$

$$\ln y=\ln\left[(\sin x)^{e^x}\right]=e^x\ln(\sin x)$$

$$\frac{y'}{y}=e^x\cdot\frac{\cos x}{\sin x}+e^x\ln(\sin x)$$

[3] $$y'=y\left[e^x\cot x+e^x\ln(\sin x)\right]$$

$$y'=(\sin x)^{e^x}e^x(\cot x+\ln(\sin x))$$

7.

(a) Use Simpson's Rule with $n = 4$ to approximate the integral $\int_1^3 \ln x \, dx$ and round your approximation to four decimal places.

$a = 1,\ b = 3,\ \Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$

$$\int_1^3 \ln x \, dx \approx \frac{3-1}{3(4)}\left[\underbrace{\ln 1}_{0} + 4\ln\tfrac{3}{2} + 2\ln 2 + 4\ln\tfrac{5}{2} + \ln 3\right]$$

$$= \frac{1}{6}\left[4\ln\tfrac{3}{2} + 2\ln 2 + 4\ln\tfrac{5}{2} + \ln 3\right]$$

[2] ≈ 1.2953

(b) Using the fact that $\frac{d}{dx}[x\ln x - x] = \ln x$, find the exact value of $\int_1^3 \ln x \, dx$ and then round this exact answer to four decimal places.

$$\int_1^3 \ln x \, dx = \left(x\ln x - x\right]_1^3 = (3\ln 3 - 3) - (1\ln 1 - 1)$$

$= 3\ln 3 - 2$ or $\ln 27 - 2$

[2] ≈ 1.2958

(c) Find the average value of $f(x) = \ln x$ on the interval $[1, 3]$. Round your answer to four decimal places.

[1] $\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{3-1}\int_1^3 \ln x \, dx = \frac{1}{2}(3\ln 3 - 2) \approx 0.6479$