CAMOSUN COLLEGE

Mathematics 100 Test #3B

Name: SOLUTIONS

Section: ______

Instructor: George Ballinger
Term: Fall, 2012

Mark: $\overline{\quad 25}$

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.

1. Find the following indefinite integrals.

(a) $\int\left(x^4+4x-4+4^x\right)dx$

$= \frac{1}{5}x^5 + 2x^2 - 4x + \frac{4^x}{\ln 4} + C$

[2]

(b) $\int \frac{\sin\theta}{\sqrt{\cos\theta}}\,d\theta = -\int \underbrace{(\cos\theta)}_{u}^{-1/2}\underbrace{(-\sin\theta)\,d\theta}_{du} = -2(\cos\theta)^{\frac{1}{2}}+C$

or $-2\sqrt{\cos\theta}+C$

[2]

(c) $\int \frac{27x^2}{(3x+2)^2}\,dx$

Let $u = 3x+2 \implies x = \frac{u-2}{3}$

$du = 3\,dx \implies dx = \frac{1}{3}du$

$= \int \frac{27\left(\frac{u-2}{3}\right)^2}{u^2}\cdot\frac{1}{3}\,du = \int \frac{(u-2)^2}{u^2}\,du = \int \frac{u^2-4u+4}{u^2}\,du$

[4]

$= \int\left(1-\frac{4}{u}+4u^{-2}\right)du = u - 4\ln|u| - \frac{4}{u} + C$

$= (3x+2) - 4\ln|3x+2| - \frac{4}{3x+2} + C$

or $3x - 4\ln|3x+2| - \frac{4}{3x+2} + C_1$ (where $C_1 = C+2$)

2. Use logarithmic differentiation to find the derivative of $y = (\ln x)^{\ln x}$.

$$\ln y = \ln (\ln x)^{\ln x} = \ln x \cdot \ln (\ln x)$$

$$\frac{y'}{y} = \ln x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \frac{1}{x} \cdot \ln (\ln x)$$

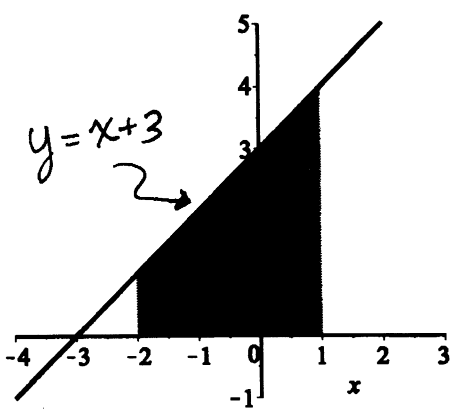
$$y' = y \left[\frac{1}{x} + \frac{\ln(\ln x)}{x} \right] = (\ln x)^{\ln x} \left(\frac{1}{x} + \frac{\ln(\ln x)}{x} \right)$$

[3]

3. Write a definite integral that represents the area of the trapezoidal region shaded below. You do not need to evaluate the integral.

$$\int_{-2}^{1} (x+3)\,dx$$

[2]



4. The velocity of an object is given by $v(t) = 4t + 2$. Find its position $s(t)$ if $s(1) = 3$.

$$s(t) = \int v(t)\,dt = \int (4t+2)\,dt = 2t^2 + 2t + C$$

[2]

$$s(1) = 3 \implies 2 + 2 + C = 3 \quad \therefore C = -1$$

$$s(t) = 2t^2 + 2t - 1$$

5. Let $f(x) = \int_{-1}^{x} t\cos t \, dt$. Evaluate

(a) $f'(1)$

$f'(x) = x\cos x$ by FTC 2

[1] $\therefore f'(1) = 1\cdot\cos 1 = \cos 1 \approx 0.5403$

(b) $f(1)$ [Hint: Use symmetry.]

[1] $f(1) = \int_{-1}^{1} t\cos t \, dt = 0$

Since $g(t) = t\cos t$ is odd $(g(-t) = -g(t))$

6. Evaluate $\int_{6}^{9} (x-6)^3 \, dx$ by using the limit of a Riemann sum definition of a definite integral:

$\Delta x = \frac{9-6}{n} = \frac{3}{n}$

[3]

$$\begin{aligned}
\int_6^9 (x-6)^3 dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(6 + i\cdot\frac{3}{n}\right)\cdot\frac{3}{n} \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left(6 + i\cdot\frac{3}{n} - 6\right)^3\cdot\frac{3}{n} \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^3\cdot\frac{3}{n} \\
&= \lim_{n\to\infty} \frac{81}{n^4} \sum_{i=1}^{n} i^3 \\
&= \lim_{n\to\infty} \frac{81}{n^4}\cdot\frac{n^2(n+1)^2}{4} \\
&= \lim_{n\to\infty} \frac{81(n^2+2n+1)}{4n^2} \\
&= \lim_{n\to\infty} \frac{81}{4}\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \\
&= \frac{81}{4}
\end{aligned}$$

7.

(a) Use Simpson's Rule with $n = 4$ to approximate the definite integral $\int_0^2 \sin^2 x\, dx$ and round your approximation to four decimal places.

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$$

$$\int_0^2 \sin^2 x\, dx \approx \frac{2-0}{3(4)}\left[f(0) + 4f(\tfrac{1}{2}) + 2f(1) + 4f(\tfrac{3}{2}) + f(2)\right]$$

$$= \frac{1}{6}\left[\sin^2 0 + 4\sin^2 \tfrac{1}{2} + 2\sin^2 1 + 4\sin^2 \tfrac{3}{2} + \sin^2 2\right]$$

[2] $$\approx 1.1904$$

(b) Using the fact that $\frac{d}{dx}\left[\frac{1}{2}(x - \sin x \cos x)\right] = \sin^2 x$, find and simplify the exact value of $\int_0^2 \sin^2 x\, dx$.

$$\int_0^2 \sin^2 x\, dx = \left[\frac{1}{2}(x - \sin x \cos x)\right]_0^2 = \frac{1}{2}(2 - \sin 2 \cos 2) - \frac{1}{2}(0 - \sin 0 \cos 0)$$

$$= 1 - \frac{1}{2}\sin 2 \cos 2$$

[2]

(Note: $1 - \frac{1}{2}\sin 2 \cos 2 \approx 1.1892$)

(c) Find the average value of $f(x) = \sin^2 x$ on the interval $[0, 2]$. Round your answer to four decimal places.

$$\text{Avg value} = \frac{1}{2-0}\int_0^2 \sin^2 x\, dx = \frac{1}{2}\left(1 - \frac{1}{2}\sin 2 \cos 2\right)$$

[1] $$\approx 0.5946$$