

The Sharp EL-531 calculator may be used on this test.  
Show all of your work in the space provided.  
The number of marks for each question is indicated in brackets.

Mark: 25

1. Find the derivative of  $y = xe^{7x^2}$ .

$$y' = x \cdot e^{7x^2} \cdot 14x + e^{7x^2} = 14x^2 e^{7x^2} + e^{7x^2} = e^{7x^2} (14x^2 + 1)$$

[2]

2. Find the following indefinite integrals.

(a)  $\int \left( \frac{3x+1}{x} \right) dx = \int \left( 3 + \frac{1}{x} \right) dx = 3x + \ln|x| + C$

[2]

(b)  $\int \frac{e^{\sqrt{t}}}{\sqrt{t}} dt = 2 \int e^{\sqrt{t}} \cdot \frac{1}{2\sqrt{t}} dt = 2e^{\sqrt{t}} + C$

[2]

(c)  $\int x\sqrt{5x+1} dx$

Let  $u = 5x+1 \Rightarrow x = \frac{u-1}{5}$   
 $du = 5dx \Rightarrow dx = \frac{1}{5} du$

[3]  $\int \frac{u-1}{5} \cdot u^{1/2} \cdot \frac{1}{5} du$   
 $= \frac{1}{25} \int (u^{3/2} - u^{1/2}) du$   
 $= \frac{1}{25} \left[ \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right] + C$   
 $= \frac{2}{125} (5x+1)^{5/2} - \frac{2}{75} (5x+1)^{3/2} + C$

OR  $= \frac{2}{375} (5x+1)^{3/2} (3(5x+1) - 5) + C$   
 $= \frac{2}{375} (5x+1)^{3/2} (15x-2) + C$

Let  $u = \sqrt{5x+1}$   
 $u^2 = 5x+1$   
 $x = \frac{u^2-1}{5} \quad dx = \frac{2}{5} u du$

OR  $\int \frac{u^2-1}{5} \cdot u \cdot \frac{2}{5} u du$   
 $= \frac{2}{25} \int (u^4 - u^2) du$   
 $= \frac{2}{25} \left[ \frac{1}{5} u^5 - \frac{1}{3} u^3 \right] + C$   
 $= \frac{2}{125} (5x+1)^{5/2} - \frac{2}{75} (5x+1)^{3/2} + C \dots$

3. Use logarithmic differentiation to differentiate  $y = (\ln x)^x$ .

$$\ln y = \ln (\ln x)^x = x \ln(\ln x)$$

$$\frac{y'}{y} = x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x)$$

[3]

$$y' = y \left( \frac{1}{\ln x} + \ln(\ln x) \right) = (\ln x)^x \left( \frac{1}{\ln x} + \ln(\ln x) \right)$$

4. Evaluate  $\int_{-2}^4 (2x+4)^3 dx$

(a) by using antiderivatives and the Fundamental Theorem of Calculus.

$$\frac{1}{2} \int_{-2}^4 (2x+4)^3 \cdot 2 dx = \frac{1}{8} (2x+4)^4 \Big|_{-2}^4 = 2592 - 0 = 2592$$

[2]

(b) by using the limit of a Riemann sum definition of a definite integral.

$$\Delta x = \frac{4 - (-2)}{n} = \frac{6}{n}$$

$$\int_{-2}^4 (2x+4)^3 dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(-2 + \frac{6i}{n}\right) \cdot \frac{6}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ 2\left(-2 + \frac{6i}{n}\right) + 4 \right]^3 \cdot \frac{6}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left( \frac{12i}{n} \right)^3 \cdot \frac{6}{n}$$

[4]

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{10368 i^3}{n^4} = \lim_{n \rightarrow \infty} \frac{10368}{n^4} \sum_{i=1}^n i^3 = \lim_{n \rightarrow \infty} \frac{10368}{n^4} \cdot \frac{n^2(n+1)}{4}$$

$$= \lim_{n \rightarrow \infty} \frac{2592(n^2+2n+1)}{n^2} = 2592 \lim_{n \rightarrow \infty} \left( 1 + \frac{2}{n} + \frac{1}{n^2} \right) = 2592$$

5. Find the derivative of  $f(x) = \log_2(8x+3)$ .

[1] 
$$f'(x) = \frac{8}{(\ln 2)(8x+3)}$$

6.

(a) Use Simpson's Rule with  $n=4$  to approximate the integral  $\int_0^2 \sqrt{4-x^2} dx$  and round your approximation to four decimal places.

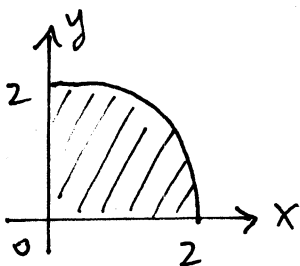
$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

$$\int_0^2 \sqrt{4-x^2} dx \approx \frac{2-0}{3(4)} [f(0) + 4f(\frac{1}{2}) + 2f(1) + 4f(\frac{3}{2}) + f(2)]$$

$$= \frac{1}{6} [2 + 4 \cdot \sqrt{\frac{15}{4}} + 2\sqrt{3} + 4\sqrt{\frac{7}{4}} + 0]$$

[2] 
$$= \frac{1}{6} [2 + 2\sqrt{15} + 2\sqrt{3} + 2\sqrt{7}] \approx 3.0836$$

(b) Sketch the region of the  $xy$ -plane whose area is represented by the integral  $\int_0^2 \sqrt{4-x^2} dx$  and then find the exact value of the integral by using a geometry formula.



$$\int_0^2 \sqrt{4-x^2} dx = \text{area of } \frac{1}{4} \text{ circle of radius 2}$$

$$= \frac{1}{4} \cdot \pi \cdot 2^2 = \pi$$

[2]

(c) Find the value of  $c$  predicted by the **Mean Value Theorem for Integrals** for the function  $f(x) = \sqrt{4-x^2}$  on the interval  $[0, 2]$ . Round your answer to four decimal places.

$$f(c) = \frac{1}{2-0} \int_0^2 \sqrt{4-x^2} dx = \frac{1}{2} \cdot \pi$$

[2]

$$\sqrt{4-c^2} = \frac{\pi}{2}$$

$$4-c^2 = \frac{\pi^2}{4}$$

$$c^2 = 4 - \frac{\pi^2}{4}$$

$$c = \sqrt{4 - \frac{\pi^2}{4}} \approx 1.2380$$