CAMOSUN COLLEGE

Mathematics 100 Test #3A

Instructor: George Ballinger
Term: Fall, 2013

Name: SOLUTIONS

Section: ______

Mark: $\frac{\quad}{25}$

> The Sharp EL-531 calculator may be used on this test.
> Show all of your work in the space provided.
> The number of marks for each question is indicated in brackets.

1. Find the derivative of $y = xe^{7x^2}$.

$$y' = x \cdot e^{7x^2} \cdot 14x + e^{7x^2} = 14x^2 e^{7x^2} + e^{7x^2} = e^{7x^2}(14x^2+1)$$

[2]

2. Find the following indefinite integrals.

(a) $\int \left(\frac{3x+1}{x}\right) dx = \int \left(3+\frac{1}{x}\right)dx = 3x + \ln|x| + C$

[2]

(b) $\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\, dt = 2\int e^{\sqrt{t}} \cdot \frac{1}{2\sqrt{t}}\, dt = 2e^{\sqrt{t}} + C$

[2]

(c) $\int x\sqrt{5x+1}\, dx$

Let $u = 5x+1 \Rightarrow x = \frac{u-1}{5}$
$du = 5dx \Rightarrow dx = \frac{1}{5}du$

[3]

$$\int \frac{u-1}{5} \cdot u^{1/2} \cdot \frac{1}{5}du$$
$$= \frac{1}{25}\int \left(u^{3/2} - u^{1/2}\right)du$$
$$= \frac{1}{25}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + C$$
$$= \frac{2}{125}(5x+1)^{5/2} - \frac{2}{75}(5x+1)^{3/2} + C$$

OR

$$= \frac{2}{375}(5x+1)^{3/2}\left(3(5x+1)-5\right) + C$$
$$= \frac{2}{375}(5x+1)^{3/2}(15x-2) + C$$

OR

Let $u = \sqrt{5x+1}$
$u^2 = 5x+1$
$x = \frac{u^2-1}{5}$ $\quad dx = \frac{2}{5}u\,du$

$$\int \frac{u^2-1}{5} \cdot u \cdot \frac{2}{5}u\,du$$
$$= \frac{2}{25}\int \left(u^4 - u^2\right)du$$
$$= \frac{2}{25}\left[\frac{1}{5}u^5 - \frac{1}{3}u^3\right] + C$$
$$= \frac{2}{125}(5x+1)^{5/2} - \frac{2}{75}(5x+1)^{3/2} + C \ldots$$

3. Use logarithmic differentiation to differentiate $y = (\ln x)^x$.

$$\ln y = \ln (\ln x)^x = x \ln(\ln x)$$

$$\frac{y'}{y} = x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x)$$

[3]

$$y' = y\left(\frac{1}{\ln x} + \ln(\ln x)\right) = (\ln x)^x\left(\frac{1}{\ln x} + \ln(\ln x)\right)$$

4. Evaluate $\int_{-2}^{4} (2x+4)^3\, dx$

(a) by using antiderivatives and the Fundamental Theorem of Calculus.

$$\frac{1}{2}\int_{-2}^{4} (2x+4)^3 \cdot 2\,dx = \frac{1}{8}(2x+4)^4 \Big|_{-2}^{4} = 2592 - 0 = 2592$$

[2]

(b) by using the limit of a Riemann sum definition of a definite integral.

$$\Delta x = \frac{4-(-2)}{n} = \frac{6}{n}$$

$$\int_{-2}^{4} (2x+4)^3 dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-2+\frac{6i}{n}\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(-2+\frac{6i}{n}\right)+4\right]^3 \cdot \frac{6}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{12i}{n}\right)^3 \cdot \frac{6}{n}$$

[4]

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{10368\, i^3}{n^4} = \lim_{n\to\infty} \frac{10368}{n^4} \sum_{i=1}^{n} i^3 = \lim_{n\to\infty} \frac{10368}{n^4} \cdot \frac{n^2(n+1)^2}{4}$$

$$= \lim_{n\to\infty} \frac{2592(n^2+2n+1)}{n^2} = 2592 \lim_{n\to\infty} \left(1 + \cancel{\frac{2}{n}}^{\,0} + \cancel{\frac{1}{n^2}}^{\,0}\right) = 2592.$$

5. Find the derivative of $f(x) = \log_2(8x+3)$.

[1] $$f'(x) = \frac{8}{(\ln 2)(8x+3)}$$

6.

(a) Use Simpson's Rule with $n = 4$ to approximate the integral $\int_0^2 \sqrt{4-x^2}\,dx$ and round your approximation to four decimal places.

$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

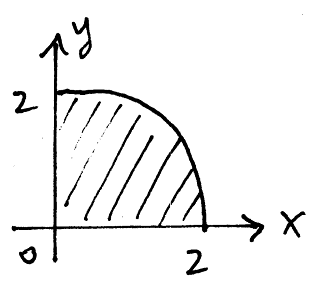
$$\int_0^2 \sqrt{4-x^2}\,dx \approx \frac{2-0}{3(4)}\left[f(0) + 4f(\tfrac{1}{2}) + 2f(1) + 4f(\tfrac{3}{2}) + f(2)\right]$$

$$= \frac{1}{6}\left[2 + 4\cdot\sqrt{\tfrac{15}{4}} + 2\sqrt{3} + 4\sqrt{\tfrac{7}{4}} + 0\right]$$

[2] $$= \frac{1}{6}\left[2 + 2\sqrt{15} + 2\sqrt{3} + 2\sqrt{7}\right] \approx 3.0836$$

(b) Sketch the region of the xy-plane whose area is represented by the integral $\int_0^2 \sqrt{4-x^2}\,dx$ and then find the exact value of the integral by using a geometry formula.

[2]



$$\int_0^2 \sqrt{4-x^2}\,dx = \text{area of } \tfrac{1}{4} \text{ circle of radius } 2$$

$$= \frac{1}{4}\cdot\pi\cdot 2^2 = \pi$$

(c) Find the value of c predicted by the **Mean Value Theorem for Integrals** for the function $f(x) = \sqrt{4-x^2}$ on the interval $[0, 2]$. Round your answer to four decimal places.

$$f(c) = \frac{1}{2-0}\int_0^2 \sqrt{4-x^2}\,dx = \frac{1}{2}\cdot\pi$$

[2] $$\sqrt{4-c^2} = \pi/2$$

$$4 - c^2 = \frac{\pi^2}{4}$$

$$c^2 = 4 - \frac{\pi^2}{4}$$

$$c = \sqrt{4 - \pi^2/4} \approx 1.2380$$