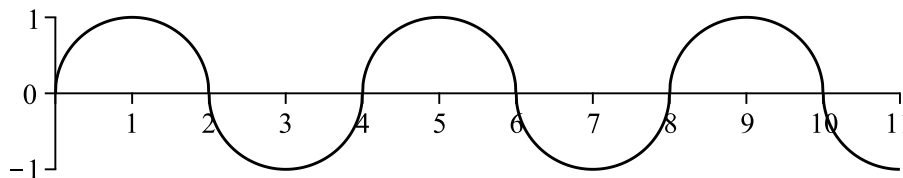


MATH 100 (Fall, 2022)
Test 3B

1. (4 marks) Evaluate $\int_1^5 6(x-1)^2 dx$ by using the limit definition of a definite integral.

2. (2 marks) Consider the periodic wave function $y = f(x)$ consisting of unit semicircles, as shown in the graph below. Evaluate the following definite integrals.



$$\int_0^2 f(x) dx = \text{---}; \quad \int_2^0 f(x) dx = \text{---}; \quad \int_0^4 f(x) dx = \text{---}; \quad \int_0^4 |f(x)| dx = \text{---}$$

3. Let $f(x) = \frac{1}{x}$.

(a) (2 marks) Use Simpson's rule with $n = 4$ to approximate $\int_1^5 f(x) dx$.

(b) (1 mark) Find the exact value of $\int_1^5 f(x) dx$.

(c) (1 mark) Find the average value of $f(x)$ on the interval $[1, 5]$.

(d) (1 mark) Find the exact value of c guaranteed to exist according to the Mean Value Theorem for Integrals for f on the interval $[1, 5]$.

4. (4 marks) Integrate $\int 9x\sqrt{3x+1} dx$.

5. (2 marks) Differentiate $f(x) = 4xe^{-5x^2}$.

6. (2 marks) Find and simplify $f'(4)$ if $f(x) = \log_2(2^x + 2)$.

7. (3 marks) Find the particular solution, $f(x)$, of the differential equation with the given initial condition.

$$f'(x) = \sec x, \quad f(\pi) = 3$$

8. (3 marks) Find the slope of the tangent line to the curve defined implicitly by the equation $y^y = x^x$ at the point $(1/4, 1/2)$. Round your answer to three decimal places.