CAMOSUN COLLEGE

Mathematics 100 Test #2A **Name:** SOLUTIONS

Instructor: George Ballinger
Term: Fall, 2009

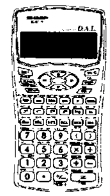
Mark: $\overline{}$ / 25

The Sharp EL-531W calculator may be used on this test.

You may not use L'Hôpital's Rule when evaluating limits.

Show all of your work in the space provided.

The number of marks for each question is indicated in brackets.



1. Find $\frac{dy}{dx}$ by implicit differentiation if $3y^3 - 7x^3 + 6x^2y = -16$.

$$9y^2\frac{dy}{dx} - 21x^2 + 6x^2\frac{dy}{dx} + 12xy = 0$$

$$(9y^2 + 6x^2)\frac{dy}{dx} = 21x^2 - 12xy$$

[2]

$$\frac{dy}{dx} = \frac{21x^2 - 12xy}{9y^2 + 6x^2} = \frac{7x^2 - 4xy}{3y^2 + 2x^2}$$

2. Find each limit and show your work. If the limit does not exist, then answer ∞ or $-\infty$ if applicable.

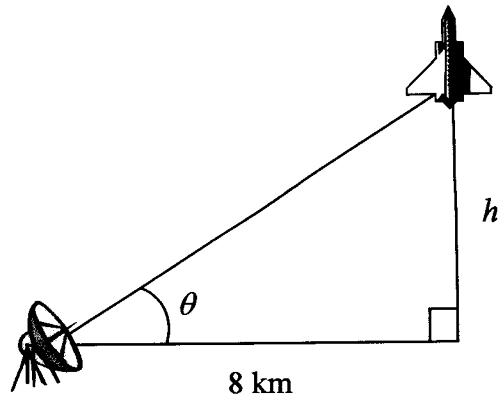
(a) $\lim\limits_{x\to\infty}\frac{x^4 + 3x^2 + 7}{x^3 + 10} = \lim\limits_{x\to\infty}\frac{x + \frac{3}{x} + \frac{7}{x^3}}{1 + \frac{10}{x^3}} = \lim\limits_{x\to\infty} x = \infty$

[1]

(b) $\lim\limits_{x\to-\infty}\frac{4x^2 - 8x + 3}{13x^2 + 2} = \lim\limits_{x\to-\infty}\frac{4 - \frac{8}{x} + \frac{3}{x^2}}{13 + \frac{2}{x^2}} = \frac{4}{13}$

[1]

3. A rocket rising vertically at a constant rate of 800 km/hr is being tracked by a radar station 8 km from the launch site. At what rate is the angle of elevation θ to the rocket from the radar station increasing (in rad/hr) when the height h of the rocket is 8 km?



$\tan\theta = \frac{h}{8}$ $\qquad \frac{dh}{dt} = 800$

$\sec^2\theta \, \frac{d\theta}{dt} = \frac{1}{8}\frac{dh}{dt}$

$\frac{d\theta}{dt} = \frac{1}{8}\cos^2\theta \, \frac{dh}{dt}$

[3] when $h = 8$, $\theta = \frac{\pi}{4}$

$\therefore \frac{d\theta}{dt} = \frac{1}{8}\left(\cos^2\frac{\pi}{4}\right)(800) = \frac{1}{8}\left(\frac{1}{2}\right)(800) = 50 \text{ rad/hr}$

4. Let $f(x) = 3x^{4/3} - 24x^{1/3}$.

(a) Find the absolute maximum and minimum values of $f(x)$ on the interval [1, 8]. Round your answers to two decimal places.

$f'(x) = 4x^{1/3} - 8x^{-2/3} = 4x^{-2/3}(x-2)$

$f'(x) = 0$ at $x = 2$ $\quad$ ($f'(x)$ is undefined at $x = 0$, but 0 is not in $[1,8]$)

[3]

endpoints $\begin{cases} f(1) = -21 \\ f(8) = 0 \end{cases}$ ← max value of $f(x)$

critical point $\{ f(2) \approx -22.68$ ← min value of $f(x)$

(b) If c is the value predicted by the Mean Value Theorem for the function f on the interval [1, 8], then calculate $f'(c)$.

[1] $f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(8) - f(1)}{8 - 1} = \frac{0 - (-21)}{7} = 3$

5. Let $f(x) = x^2 + 4\sin x$ on the interval $[0, \pi/2]$. Find the open interval(s) on which $f(x)$ is concave upward and the open interval(s) on which $f(x)$ is concave downward and find the coordinates of all inflection points on the graph of $f(x)$.

[3]

$f'(x) = 2x + 4\cos x$

$f''(x) = 2 - 4\sin x$

$f''(x) = 0 \Rightarrow 2 - 4\sin x = 0$

$\Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$

$f(\frac{\pi}{6}) = (\frac{\pi}{6})^2 + 4\sin\frac{\pi}{6} = \frac{\pi^2}{36} + 2$

	$(0, \frac{\pi}{6})$	$(\frac{\pi}{6}, \frac{\pi}{2})$
Sign of f''	+	−
f	$\smile$	$\frown$

f is concave upward on $(0, \frac{\pi}{6})$

f is concave downward on $(\frac{\pi}{6}, \frac{\pi}{2})$

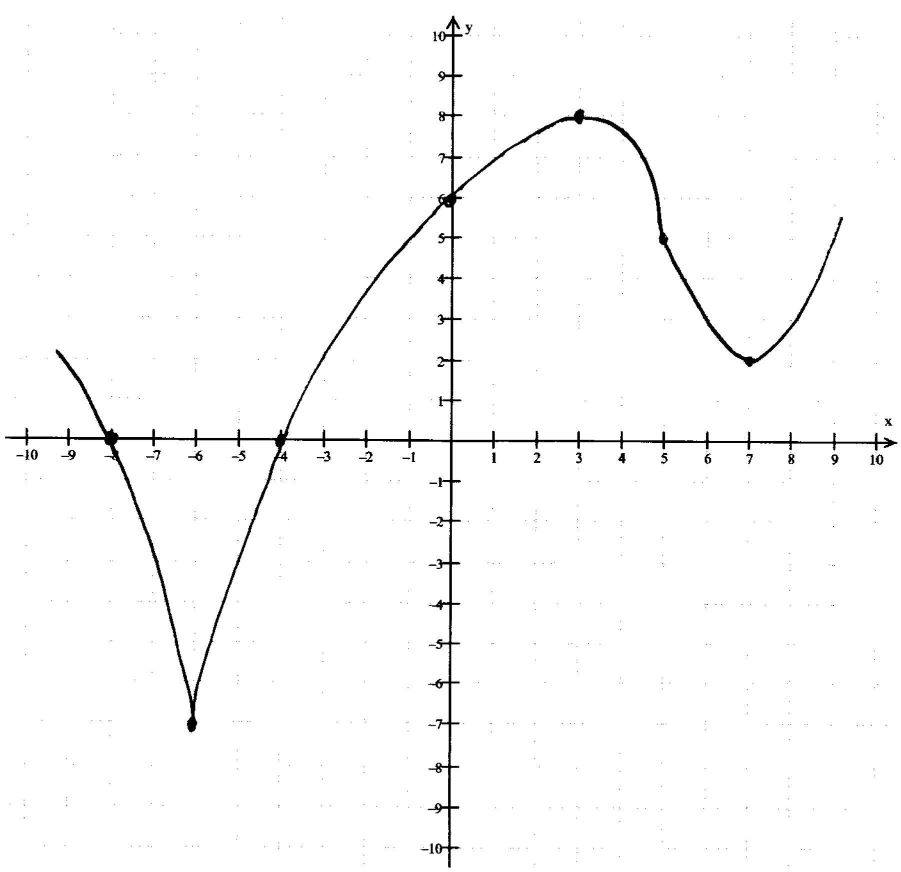
f has inflection point at $(\frac{\pi}{6}, \frac{\pi^2}{36} + 2)$

$= (\frac{\pi}{6}, 2.274)$

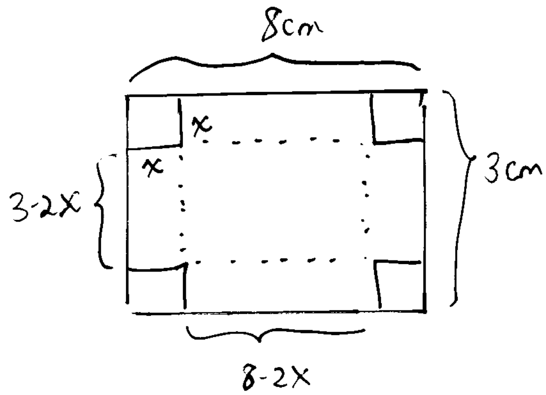
6. Sketch the graph of a continuous function f that clearly satisfies all of the following characteristics.

f has x-intercepts at $x = -8$ and $x = -4$	$f'(x) > 0$ on $(-6, 3)$ and $(7, \infty)$
f has a y-intercept at $y = 6$	$f'(x) < 0$ on $(-\infty, -6)$ and $(3, 7)$
$f(-6) = -7$, $f(3) = 8$, $f(5) = 5$ and $f(7) = 2$	$f''(x) > 0$ on $(5, \infty)$
$f'(-6)$ is undefined, $f'(3) = 0$ and $f'(7) = 0$	$f''(x) < 0$ on $(-\infty, -6)$ and $(-6, 5)$

[3]



7. A open box is to be made by cutting square pieces from the corners of a 3 cm × 8 cm sheet of material and folding up the sides. Find the maximum volume that such a box can have and use the Second Derivative Test to confirm that your answer is a maximum.



[4]

$$V = x(8-2x)(3-2x) \qquad \text{for } 0<x<3$$
$$= 4x^3 - 22x^2 + 24x$$

$$V'(x) = 12x^2 - 44x + 24$$
$$= 4(3x^2 - 11x + 6)$$
$$= 4(3x-2)(x-3)$$

$V'(x) = 0$ when $x = \frac{2}{3}$ or $\underbrace{x = 3}_{\text{too big}}$

$$V''(x) = 24x - 44$$
$$V''\left(\tfrac{2}{3}\right) = 24\left(\tfrac{2}{3}\right) - 44 = -28 < 0 \quad \therefore \text{ Max by SDT}$$

$$V\left(\tfrac{2}{3}\right) = \tfrac{2}{3}\left(8 - 2\cdot\tfrac{2}{3}\right)\left(3 - 2\cdot\tfrac{2}{3}\right) = \tfrac{200}{27}\,cm^3 \approx 7.41\ cm^3$$

8. Suppose f is a differentiable function and the slope of the tangent line at a point $(-2, 1)$ on its graph is 9. If Newton's Method were used with an initial estimate $x_1 = -2$ to approximate a zero of $f(x)$, then compute the second estimate x_2.

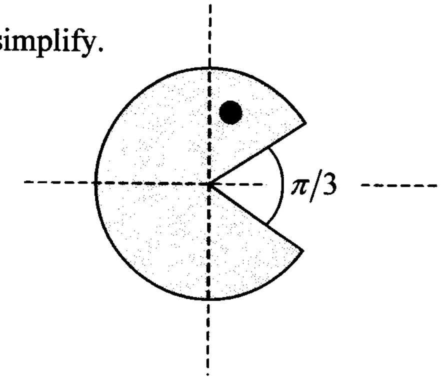
$$f(-2) = 1 \qquad f'(-2) = 9$$

[1]

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -2 - \frac{1}{9} = \frac{-19}{9} \approx -2.111$$

9. Pac-Man is in the shape of a circle of radius r with his mouth opened at an angle $\pi/3$.

(a) Express the area A of Pac-Man as a function of his radius r and simplify.



[1]

$$A = \left(\frac{2\pi - \frac{\pi}{3}}{2\pi}\right)\pi r^2 = \frac{5}{6}\pi r^2$$

(b) If the radius r is measured to be 1.2 cm with a possible error of 0.03 cm, then using differentials approximate the possible propagated error in calculating the area of Pac-Man. Round your answer to three decimal places.

$r = 1.2$ cm

$dr = \pm 0.03$cm

$$dA = \frac{5}{3}\pi r \, dr$$

[2]

$$= \frac{5}{3}\pi (1.2)(\pm 0.03)$$

$$= \pm 0.188 \text{ cm}^2$$