CAMOSUN COLLEGE

Mathematics 100 Test #2A **Name:** SOLUTIONS

Instructor: George Ballinger **Section:** ____________

Term: Fall, 2011

Mark: $\frac{\quad}{25}$

The Sharp EL-531 calculator may be used on this test.
You may not use L'Hôpital's Rule when evaluating limits.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.

1. Find the limit and show your work. If the limit does not exist, then answer ∞ or $-\infty$ if applicable.

$$\lim_{x\to-\infty}\frac{8x^2+7-2x^3+11x}{x-7x^3+x^2-3}$$

[2]

$$=\lim_{x\to-\infty}\frac{\frac{8}{x}+\frac{7}{x^3}-2+\frac{11}{x^2}}{\frac{1}{x^2}-7+\frac{1}{x}-\frac{3}{x^3}}=\frac{-2}{-7}=\frac{2}{7}$$

2. Use implicit differentiation to find $\frac{dy}{dx}$ for the following curve and simplify your answer.

$$2x^3+10xy^2-7x^2=5y^4+13$$

$$6x^2+10x(2yy')+10y^2-14x=20y^3y'$$

$$(20xy-20y^3)y'=14x-6x^2-10y^2$$

[3]

$$y'=\frac{14x-6x^2-10y^2}{20xy-20y^3}=\frac{2(7x-3x^2-5y^2)}{20y(x-y^2)}=\frac{7x-3x^2-5y^2}{10y(x-y^2)}$$

3. If Newton's Method were used to approximate a zero of $f(x)=x^3+3x^2+1$ using an initial approximation of $x_1=-3$, then compute the next approximation x_2.

$$f'(x)=3x^2+6x$$

[2]

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=-3-\frac{f(-3)}{f'(-3)}=-3-\frac{1}{9}=\frac{-28}{9}\approx-3.1111$$

4. Consider the function $f(x)=\dfrac{-3x^2+3}{x^2+1}$. Its first two derivatives are $f'(x)=\dfrac{-12x}{(x^2+1)^2}$ and $f''(x)=\dfrac{12(3x^2-1)}{(x^2+1)^3}$.

(a) Find the coordinates of all x and y-intercepts and the equations of all asymptotes.

[2]

$f(0)=3$

$\therefore$ y-int is $(0,3)$

$f(x)=0$

$\Rightarrow -3x^2+3=0$

$\Rightarrow -x^2+1=0$

$\Rightarrow x^2=1$

$\Rightarrow x=\pm 1$

$\therefore (\pm 1, 0)$ are x-int

No V.A.

$$\lim_{x\to\pm\infty}\frac{-3x^2+3}{x^2+1}=\lim_{x\to\pm\infty}\frac{-3+\frac{3}{x^2}}{1+\frac{1}{x^2}}=-3$$

$\therefore y=-3$ is HA

(b) Find the intervals on which f is increasing or decreasing and find the coordinates of all critical points. Classify each critical point as a relative maximum, relative minimum or neither

[2]

$f'(x)=0$ at $x=0$ where $f(0)=3$

Intervals	$(-\infty,0)$	$(0,\infty)$
f'	$+$	$-$
f	$\nearrow$	$\searrow$

f is decreasing on $(0,\infty)$

f is increasing on $(-\infty,0)$

By FDT f has a relative max at $(0,3)$ of 3.

(c) Find the intervals on which the graph of f is concave upward or concave downward and find the coordinates of all inflection points.

[2]

$f''(x)=0$ at $x=\pm\frac{1}{\sqrt{3}}$ where $f(\pm\frac{1}{\sqrt{3}})=\frac{3}{2}$

Intervals	$(-\infty,-\frac{1}{\sqrt{3}})$	$(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})$	$(\frac{1}{\sqrt{3}},\infty)$
f''	$+$	$-$	$+$
f	$\cup$	$\cap$	$\cup$

f is concave up on $(-\infty,-\frac{1}{\sqrt{3}})$ and $(\frac{1}{\sqrt{3}},\infty)$

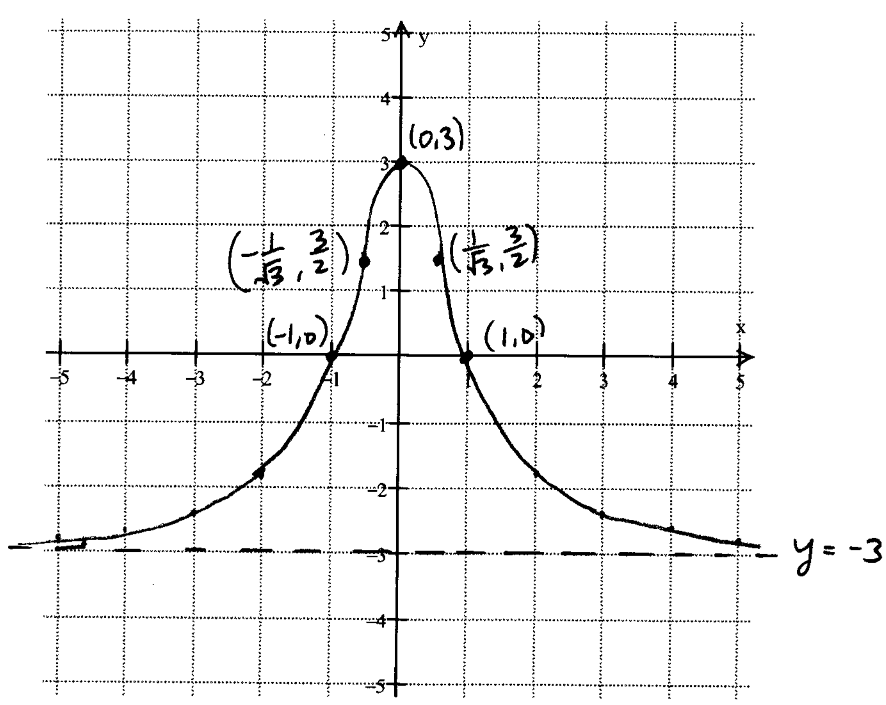
f is concave down on $(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})$

$(\pm\frac{1}{\sqrt{3}},\frac{3}{2})$ are inflection points

$\approx (\pm 0.58, 1.5)$

(d) Use the above information and possibly extra points to sketch the graph of the function. Clearly label **all** intercepts, asymptotes, critical points and inflection points on your graph.

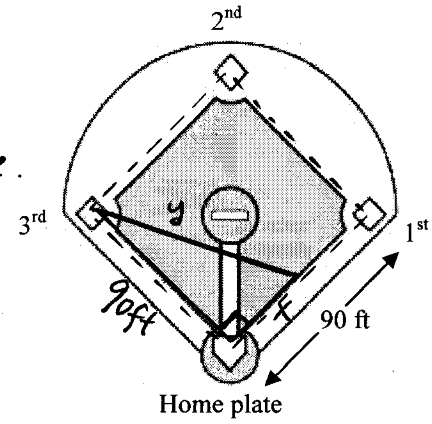
[2]



Extra Points

x	y
± 2	-1.8
± 3	-2.4
± 4	-2.6
± 5	-2.8

5. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs from home plate toward first base with a speed of 24 ft/sec. At what rate is his distance from third base increasing when he is halfway to first base? Round your answer to three decimal places.



Let y be distance between batter and 3rd base and let x be distance between batter and home plate.

Equation : $x^2 + 90^2 = y^2$

Given : $\frac{dx}{dt} = 24$ ft/sec.

[4] Find : $\frac{dy}{dt}$ when $x = 45$ ft

$$\frac{d}{dt}\left[x^2+90^2\right] = \frac{d}{dt}\left[y^2\right]$$

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{45}{\sqrt{90^2+45^2}} \cdot 24 \approx 10.733 \text{ ft/sec}$$

The distance from 3rd base is increasing at a rate of 10.733 ft/sec.

6. Let $g(\theta) = 2\theta + 4\cos\theta$. Find the absolute minimum and maximum values of g on the interval $[0, 2\pi]$. Round your answers to three decimal places.

$$g'(\theta) = 2 - 4\sin\theta$$

$$g'(\theta) = 0 \Rightarrow \sin\theta = \frac{2}{4} = \frac{1}{2} \Rightarrow \theta = \underbrace{\frac{\pi}{6} \text{ or } \frac{5\pi}{6}}_{\text{Critical \#'s}}$$

[3]

$g(0) = 4$

$g(2\pi) = 4\pi + 4 \approx 16.566$ ← MAX

$g(\frac{\pi}{6}) = \frac{\pi}{3} + 2\sqrt{3} \approx 4.511$

$g(\frac{5\pi}{6}) = \frac{5\pi}{3} - 2\sqrt{3} \approx 1.772$ ← MIN

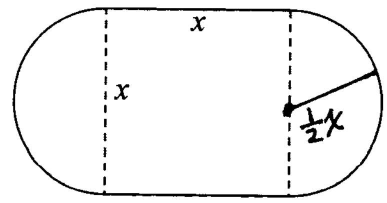
The max value of g is 16.566 and the min value of g is 1.772.

7. An ice rink is in the shape of a square with a semicircle on opposite ends as shown in the figure.

(a) Express the area A of the ice rink as a function of its width x and simplify.

[1]

$$A = x^2 + \pi\left(\tfrac{1}{2}x\right)^2$$
$$= \left(1 + \tfrac{\pi}{4}\right)x^2$$



(b) If the width of the ice rink is measured to be 85 ft with a possible error of 0.5ft, then using **differentials** approximate the possible propagated error in calculating the area of the ice rink. Round your answer to one decimal place.

[2]

$$\Delta A \approx dA = 2\left(1+\tfrac{\pi}{4}\right)x\,dx = 2\left(1+\tfrac{\pi}{4}\right)(85\text{ft})(\pm 0.5\text{ft}) \approx \pm 151.8\text{ft}^2$$