CAMOSUN COLLEGE

Mathematics 100 Test #2A **Name:** SOLUTIONS

Instructor: George Ballinger **Section:** ____________

Term: Fall, 2012

The Sharp EL-531 calculator may be used on this test.

Show all of your work in the space provided.

The number of marks for each question is indicated in brackets.

Mark: $\frac{\quad}{25}$

1. Find dy/dx by using implicit differentiation. Simplify your answer.

$$4y^5 - 3x^6y^2 + 6x^2 = 7 + 2\cos y$$

$$20y^4\frac{dy}{dx} - 3x^6\left(2y\frac{dy}{dx}\right) - 18x^5y^2 + 12x = -2\sin y\frac{dy}{dx}$$

$$\left(20y^4 - 6x^6y + 2\sin y\right)\frac{dy}{dx} = 18x^5y^2 - 12x$$

[4]

$$\frac{dy}{dx} = \frac{18x^5y^2 - 12x}{20y^4 - 6x^6y + 2\sin y} = \frac{9x^5y^2 - 6x}{10y^4 - 3x^6y + \sin y}$$

$$\text{or } \frac{3x(3x^4y^2 - 2)}{10y^4 - 3x^6y + \sin y}$$

2. If Newton's Method were used to approximate a zero of $f(x) = x^2 - 98$ using an initial approximation of $x_1 = 10$, then compute the next approximation x_2.

$$f'(x) = 2x$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 10 - \frac{f(10)}{f'(10)} = 10 - \frac{2}{20} = 10 - \frac{1}{10} = \frac{99}{10} \text{ or } 9.9$$

[2]

3. Consider the function $f(x) = \dfrac{4x+4}{x^2}$. Its first two derivatives are $f'(x) = \dfrac{-4x-8}{x^3}$ and $f''(x) = \dfrac{8x+24}{x^4}$.

[9]

(a) Find the coordinates of all intercepts and the equations of all asymptotes.

$f(0)$ undefined

$\therefore$ no y-int

$f(x) = \dfrac{4(x+1)}{x^2}$

$f(x) = 0$ at $x = -1$

$\therefore (-1, 0)$ is x-int

$x = 0$ is V.A

$\lim_{x \to \pm\infty} f(x)$

$= \lim_{x \to \pm\infty} \left(\dfrac{4}{x} + \dfrac{4}{x^2}\right) = 0$

$\therefore y = 0$ is HA

(b) Find the intervals on which f is increasing or decreasing and find the coordinates of all critical points. Classify each critical point as a relative maximum, relative minimum or neither.

$f'(x) = \dfrac{-4(x+2)}{x^3}$

$f'(x) = 0$ at $x = -2$ $\quad f(-2) = -1$ $\quad \therefore (-2,-1)$ is critical point

$f'(x)$ is undefined at $x = 0$ (discontinuity of f)

Intervals	$(-\infty, -2)$	$(-2, 0)$	$(0, \infty)$
f'	$-$	$+$	$-$
f	↘	↗	↘

f is decreasing on $(-\infty, -2)$ and $(0, \infty)$ and increasing on $(-2, 0)$

By FDT f has local min at $(-2, -1)$

(c) Find the intervals on which the graph of f is concave upward or concave downward and find the coordinates of all inflection points.

$f''(x) = \dfrac{8(x+3)}{x^4}$

$f''(x) = 0$ at $x = -3$ $\quad f(-3) = \dfrac{-8}{9}$

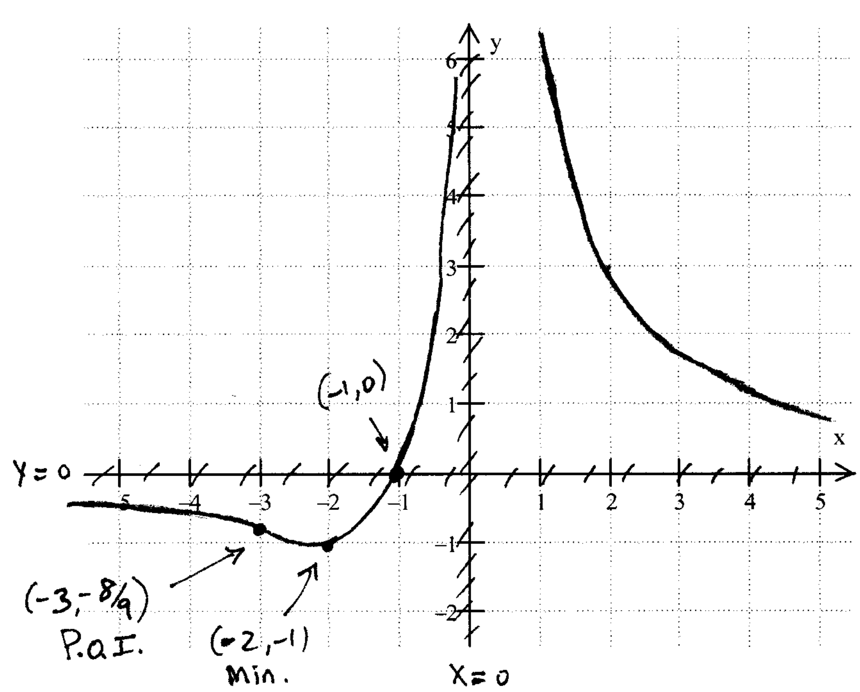
$f''(x)$ is undefined at $x = 0$ (discontinuity of f)

Intervals	$(-\infty, -3)$	$(-3, 0)$	$(0, \infty)$
f''	$-$	$+$	$+$
f	∩	∪	∪

Graph of f is concave down on $(-\infty, -3)$ and Concave up on $(-3, 0)$ and $(0, \infty)$.

$(-3, -\frac{8}{9})$ is a P.O.I.

(d) Use the above information and possibly extra points to sketch the graph of the function. Clearly label **all** intercepts, asymptotes, critical points and inflection points on your graph.



4. Find the **absolute** maximum and minimum values of the following function on the interval [1, 2].

$$f(x) = x\sqrt{4-x^2}$$

$$f'(x) = x \cdot \frac{1}{2\sqrt{4-x^2}} \cdot (-2x) + \sqrt{4-x^2} = \frac{-x^2}{\sqrt{4-x^2}} + \frac{4-x^2}{\sqrt{4-x^2}}$$

[5]

$$= \frac{4-2x^2}{\sqrt{4-x^2}}$$

$f'(x) = 0$ when $4-2x^2 = 0$

$x^2 = 2$

$x = \sqrt{2}$ or $-\sqrt{2}$ not in $(1,2)$

$f'(x)$ undef. at $x = \pm 2$ not in $(1,2)$

$x = \sqrt{2}$ is critical # in $(1,2)$

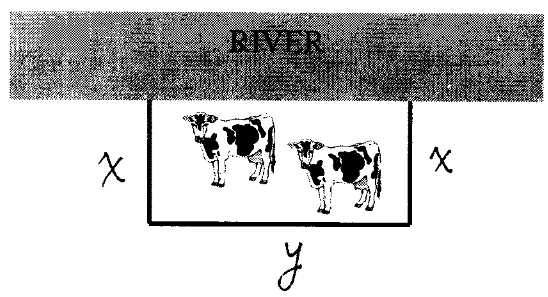
$f(1) = \sqrt{3}$

$f(2) = 0$ ← MIN

$f(\sqrt{2}) = 2$ ← MAX

The max value of f is 2 and the min value is 0.

5. A farmer wishes to fence a rectangular pasture for his herd of cows. One side of the pasture, which is to be adjacent to a river, requires no fencing. The total area of the pasture must be 125,000 m^2. What dimensions will require the least amount of fencing? Be sure to verify that your solution does give the absolute minimum.



Min $P = 2x + y$

Subject to $xy = 125000 \Rightarrow y = \frac{125000}{x}$

$P = 2x + \frac{125000}{x} = 2x + 125000x^{-1}$

domain $x > 0$

$$P' = 2 - 125000x^{-2} = 2 - \frac{125000}{x^2} = \frac{2x^2 - 125000}{x^2} = \frac{2(x^2 - 62500)}{x^2}$$

[5]

$$= \frac{2(x-250)(x+250)}{x^2}$$

$P'(x) = 0$ at $x = 250$

(and $x = -250$ ← not in domain)

When $x = 250$, $y = \frac{125000}{250} = 500$

Use FDT or SDT to verify $x = 250$ minimizes P

FDT

Intervals	$(0, 250)$	$(250, \infty)$
P'	$-$	$+$
P	↘	↗

$\therefore$ min at $x = 250$

OR

SDT $P'' = 250\,000x^{-3}$

$P''(250) = \frac{2}{125} > 0$ ✓

$\therefore$ min at $x = 250$

Dimensions should be 500 m (along river) by 250 m.