CAMOSUN COLLEGE

Mathematics 100 Test #2B Name: SOLUTIONS

Instructor: George Ballinger **Section:** ____________

Term: Fall, 2013

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.

Mark: $\frac{\quad}{25}$

1. Find the limit and show your work. If the limit does not exist, then answer ∞ or $-\infty$ if applicable.

(a) $\lim\limits_{x\to\infty} \dfrac{4+5x-2x^2}{3-11x+7x^2} = \lim\limits_{x\to\infty} \dfrac{\cancel{\frac{4}{x^2}}^{\,0} + \cancel{\frac{5}{x}}^{\,0} - 2}{\cancel{\frac{3}{x^2}}_{\,0} - \cancel{\frac{11}{x}}_{\,0} + 7} = -\dfrac{2}{7}$

[2]

(b) $\lim\limits_{x\to-\infty} \dfrac{5x^{4/3}-8x^{1/3}+1}{2x^{1/3}+5} = \lim\limits_{x\to-\infty} \dfrac{5x - 8 + \cancel{\frac{1}{x^{1/3}}}^{\,0}}{2 + \cancel{\frac{5}{x^{1/3}}}_{\,0}} = -\infty$

[2]

2. If $y = \dfrac{3}{2x+1}$, then find the differential dy.

$y = 3(2x+1)^{-1}$

$dy = -3(2x+1)^{-2}(2)\,dx = \dfrac{-6\,dx}{(2x+1)^2}$

[2]

3. Use implicit differentiation to find $\dfrac{dy}{dx}$ for the curve $y = \sin xy$.

$y' = \cos xy \cdot (xy' + y) = x(\cos xy)\,y' + y\cos xy$

$y'(1 - x\cos xy) = y\cos xy$

$y' = \dfrac{y\cos xy}{1 - x\cos xy}$

[3]

4. Suppose f is a differentiable function satisfying $f(10) = 4$ and $f'(10) = -5$. Use a linear approximation to approximate $f(10.02)$.

$$f(x) \approx f(c) + f'(c)(x-c)$$

$$f(10.02) \approx f(10) + f'(10)(10.02-10) = 4 + (-5)(0.02) = 3.9$$

[2]

5. Let $f(x) = \dfrac{2x^4 + x^2 + 2}{x^4 + 1}$. Its derivative is $f'(x) = \dfrac{-2x^5 + 2x}{(x^4+1)^2}$. Find the open intervals on which f is increasing or decreasing and find the relative extrema of f.

$$f'(x) = \frac{-2x(x^4-1)}{(x^4+1)^2} = \frac{-2x(x^2+1)(x^2-1)}{(x^4+1)^2} = \frac{-2x(x^2+1)(x-1)(x+1)}{(x^4+1)^2}$$

$f'(x) = 0$ at $x = 0, 1, -1$ (critical #'s).

$f(0) = 2$, $f(1) = \frac{5}{2}$, $f(-1) = \frac{5}{2}$

[4]

Intervals	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of f'	+	−	+	−
f	↗	↘	↗	↘

f is increasing on $(-\infty, -1)$ and $(0, 1)$

f is decreasing on $(-1, 0)$ and $(1, \infty)$

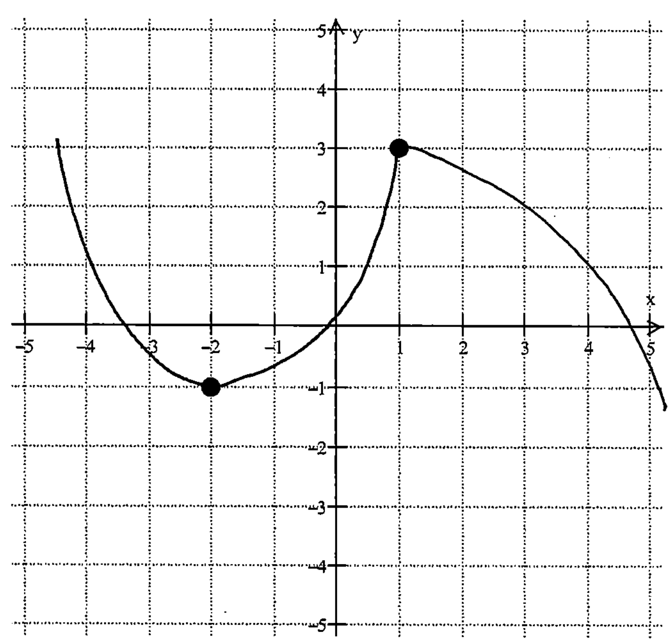
By FDT f has a relative max. at $x = 1$ and $x = -1$ of $5/2$ and f has a relative min. at $x = 0$ of 2.

6. Sketch the graph of a **continuous** function f that passes through the indicated points and that satisfies all of the following properties. Your graph should clearly show the increasing, decreasing and concave structure of f.

$f'(-2) = 0$
$f'(1)$ is undefined

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
Sign of $f'(x)$	−	+	−
Sign of $f''(x)$	+	+	−
f	↘	↗	↘

[2]



7. Use Newton's Method with an initial estimate of $x_1 = 1$ to approximate a solution of the equation $x - \cos x = 0$. Round your answer to as many decimal places as your Sharp EL-531 calculator will give you and list the values of all your estimates x_1, x_2, x_3, etc.

(calc. in RAD mode)

Let $f(x) = x - \cos x$

$f'(x) = 1 + \sin x$

$$x_{n+1} = x_n - \frac{x_n - \cos x_n}{1 + \sin x_n}$$

$x_1 = 1$

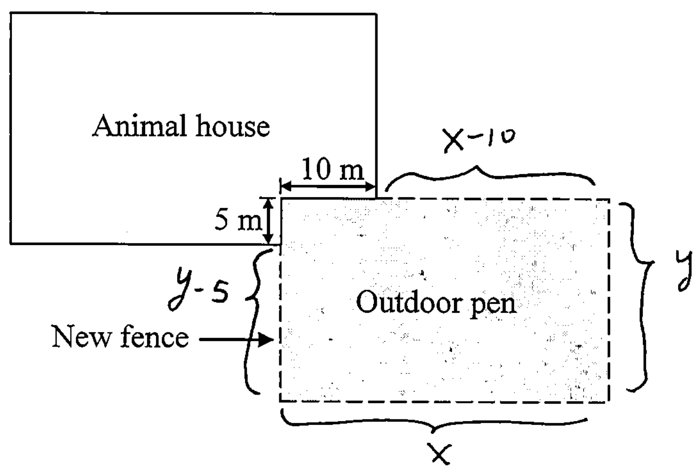
$x_2 = 0.750363867$

$x_3 = 0.739112890$

$x_4 = 0.739085133$

[3] $x_5 =$ same

8. A zookeeper needs to add a rectangular outdoor pen to an animal house with a corner notch as show in the figure. If 85 m of new fence is available, what dimensions of the pen will maximize its area? No fence will be used along the walls of the animal house. Use calculus (derivatives) to find the optimal dimensions and be sure to verify that the dimensions you find do in fact maximize the area of the pen.



Maximize Area $A = xy$

subject to $x + (x-10) + y + (y-5) = 85$

$2x + 2y = 100$

$x + y = 50$

$y = 50 - x$

$A = x(50-x) = 50x - x^2$

domain: $x \geq 10$

$y \geq 5 \Rightarrow 50 - x \geq 5$

$x \leq 45$

$\therefore [10, 45]$

[5]

$A' = 50 - 2x = 0$ at $\underbrace{x = 25}_{\text{critical \# in } (10,45)}$

$y = 50 - 25 = 25$

Any one of these 3 methods prove $x = 25$ maximizes A:

① Endpoints of closed interval

$A(10) = 400$

$A(25) = 625 \leftarrow$ MAX

$A(45) = 225$

② FDT

Intervals	$(10,25)$	$(25,45)$
A'	$+$	$-$
A	$\nearrow$	$\searrow$

$\therefore$ MAX

③ SDT

$A'' = -2$

$A''(25) = -2 < 0$

$\frown$ $\therefore$ MAX

The outdoor pen should be a square measuring 25m by 25m.